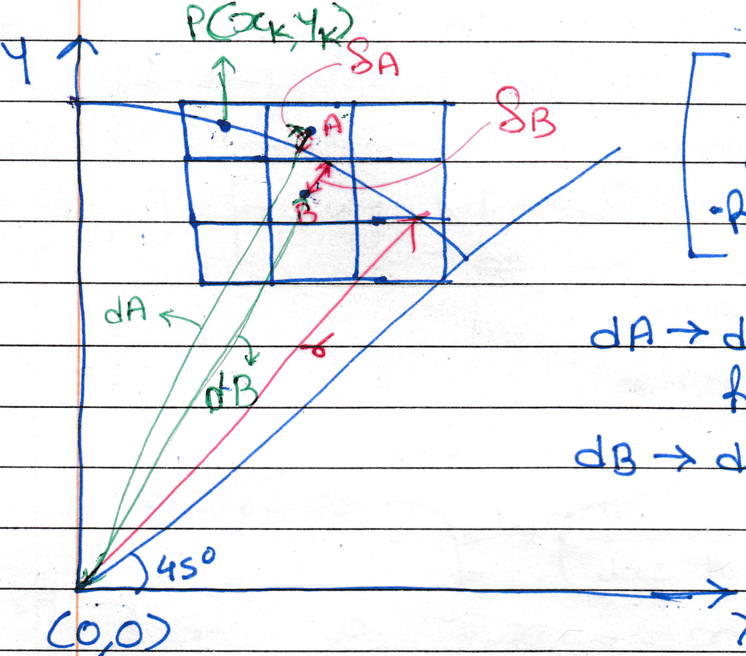
Bresenham Circle Algo

[Derivation for decision parameter]



[δ_A & δ_B are the distances of pixels A & B from true circle]

dA → distance of ~~point~~ pixel A from centre of circle (0,0)

dB → distance of ~~point~~ pixel B from centre of circle (0,0)

Main logic behind this algo.

- Assume that, ~~poin~~ pixel $P(x_k, y_k)$ is a point on circle.
- Now we have to choose either pixel A or pixel B [if $\delta_A < \delta_B$ then pixel A is selected otherwise pixel B is selected]
- Co-ordinates of pixel A → $(x_k + 1, y_k)$
- Co-ordinates of pixel B → $(x_k + 1, y_k - 1)$
- Distances of pixels A & B from origin [centre of circle] can be calculated as :- (Using euclidean distance) :-

$$dA = \sqrt{(x_k + 1 - 0)^2 + (y_k - 0)^2}$$

$$dA = \sqrt{(x_k + 1)^2 + y_k^2} \quad \text{—} (1)$$

$$dB = \sqrt{(x_k+1-0)^2 + (y_k-1-0)^2}$$

$$dB = \sqrt{(x_k+1)^2 + (y_k-1)^2} \quad \text{—— (2)}$$

- Distances of pixels A & B from true circle can be calculated as :-

$\delta_A = dA - r$ ← (will be always +ve because pixel A is outside true circle)

&

$\delta_B = dB - r$ ← (will be always −ve because pixel B is inside true circle)

- To avoid square root from the derivation of decision parameter, we will define δ_A & δ_B as follows :

$$\delta_A = dA^2 - r^2 \quad \text{—— (3)} \rightarrow \text{always +ve}$$

$$\delta_B = dB^2 - r^2 \quad \text{—— (4)} \rightarrow \text{always −ve}$$

putting eq^n (1) & (2) in eq^n (3) & (4) respectively

$$\delta_A = (x_k+1)^2 + y_k^2 - r^2 \quad \text{—— (5)}$$

$$\delta_B = (x_k+1)^2 + (y_k-1)^2 - r^2 \quad \text{—— (6)}$$

⇒ Decision parameter, d_k can be defined as :-

$$\boxed{d_k = \delta_A + \delta_B} \quad \text{—— (A)}$$ putting eq^n (5) & (6)

$$d_k = \underbrace{(x_k+1)^2 + y_k^2 - r^2}_{d_A} + \underbrace{(x_k+1)^2 + (y_k-1)^2 - r^2}_{d_B}$$

$$\therefore \quad d_k = 2(x_k+1)^2 + y_k^2 + (y_k-1)^2 - 2r^2 \quad \text{—(7)}$$

By replacing K by K+1, we can find next decision parameter d_{k+1} as :-

$$d_{k+1} = 2(x_{k+1}+1)^2 + y_{k+1}^2 + (y_{k+1}-1)^2 - 2r^2$$

Remember,

x_{k+1} & y_{k+1} are co-ordinates of the pixel which is going to be printed after current pixel $P(x_k, y_k)$

Now,

$$d_{k+1} - d_k = 2(x_{k+1}+1)^2 + y_{k+1}^2 + (y_{k+1}-1)^2 - \cancel{2r^2}$$
$$- \left[2(x_k+1)^2 + y_k^2 + (y_k-1)^2 - \cancel{2r^2}\right]$$

$$= 2(x_{k+1}+1)^2 + y_{k+1}^2 + (y_{k+1}-1)^2$$
$$- 2(x_k+1)^2 - y_k^2 - (y_k-1)^2 \quad \text{—(8)}$$

Regardless which point from A & B is selected as next point, in both cases :→ $\boxed{x_{k+1} = x_k + 1}$

Because for both pixel A & B, x co-ordinate is incremented by 1.

$\therefore$ Put $x_{k+1} = x_k + 1$ in eq^n 8

$$d_{k+1} - d_k = 2(x_k+1+1)^2 + y_{k+1}^2 + (y_{k+1}-1)^2 - 2(x_k+1)^2 - y_k^2 - (y_k-1)^2$$

$$= 2(x_k+2)^2 + y_{k+1}^2 + (y_{k+1}-1)^2 - 2(x_k+1)^2 - y_k^2 - (y_k-1)^2$$

$$= 2(\cancel{x_k^2} + 4x_k + 4) + y_{k+1}^2 + (y_{k+1}^2 - 2y_{k+1} + 1) - 2(\cancel{x_k^2} + 2x_k + 1) - y_k^2 - (y_k^2 - 2y_k + 1)$$

$$= 8x_k + 8 + 2y_{k+1}^2 - 2y_{k+1} + 1 - 4x_k - 2 - 2y_k^2 + 2y_k - 1$$

$$d_{k+1} - d_k = 4x_k + 2(y_{k+1}^2 - y_k^2) - 2(y_{k+1} - y_k) + 6 \quad \text{— 9}$$

writing eq^n A again,

$$d_k = \underbrace{\delta_A}_{\text{always +ve}} + \underbrace{\delta_B}_{\text{always -ve}} \quad \text{— A}$$

Now,

⇒ Case - 1 :- Point A is selected

~~ie.~~ → if $\delta_A < \delta_B$

ie → $d_k < 0$ because & $d_k = \underset{\text{always +ve}}{\delta_A} + \underset{\text{always -ve}}{\delta_B}$

→ $x_{k+1} = x_k + 1$ already substituted

& $y_{k+1} = y_k$ put this value in eq^n (9)

$$\therefore d_{k+1} - d_k = 4x_k + 2(\cancel{y_k^2} - \cancel{y_k^2}) - 2(\cancel{y_k} - \cancel{y_k}) + 6$$

$$\therefore \boxed{d_{k+1} = d_k + 4x_k + 6} \text{ — (I)}$$

⇒ Case 2 :- Point B is selected

→ if $\delta_A \geq \delta_B$

→ $d_k \geq 0$

→ $x_{k+1} = x_k + 1$ already substituted

& $y_{k+1} = y_k - 1$ Put this in eq^n (9)

$$d_{k+1} - d_k = 4x_k + 2[(y_k - 1)^2 - y_k^2] - 2[(\cancel{y_k} - 1) - \cancel{y_k}] + 6$$

$$= 4x_k + 2[(\cancel{y_k^2} - 2y_k + 1) - \cancel{y_k^2}] + 8$$

$$= 4x_k - 4y_k + 10$$

$$d_{k+1} - d_k = 4(x_k - y_k) + 10$$

$$\therefore \boxed{d_{k+1} = d_k + 4(x_k - y_k) + 10} \quad \text{II}$$

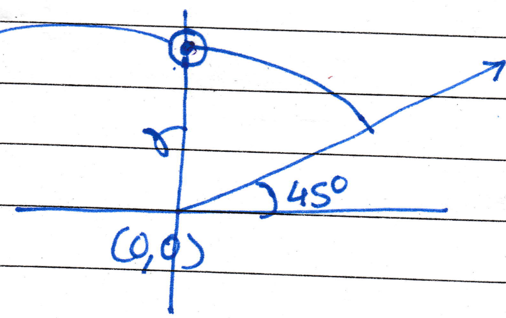
⇒ To find initial decision parameter d_k :-

At the first ~~poi~~ pixel in first octant,

$x = 0, \; y = r$

at initial point,

$\therefore x_k = 0, \; y_k = r$



Put these values in eq^n 7

$$d_k = 2(0+1)^2 + r^2 + (r-1)^2 - 2r^2$$

$$= 2 + \cancel{r^2} + \cancel{r^2} - 2r + 1 - \cancel{2r^2}$$

$$\boxed{d_k = 3 - 2r} \quad \text{III}$$

Initial decision parameter