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DATE :

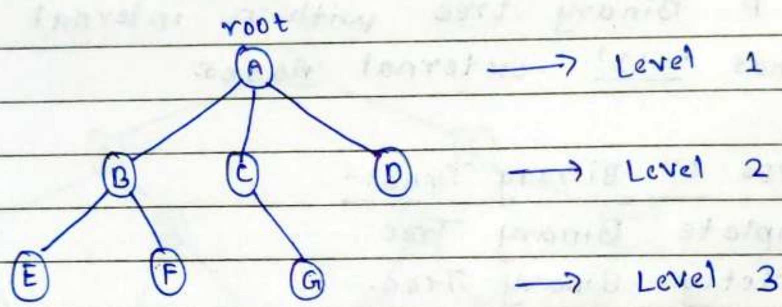
12/02/24 MON.

UNIT NO. 2 (TWO): TREES

- **TREE:-**

A Tree is an abstract model of a hierarchical structure that consists of nodes with a parent-child relationship.

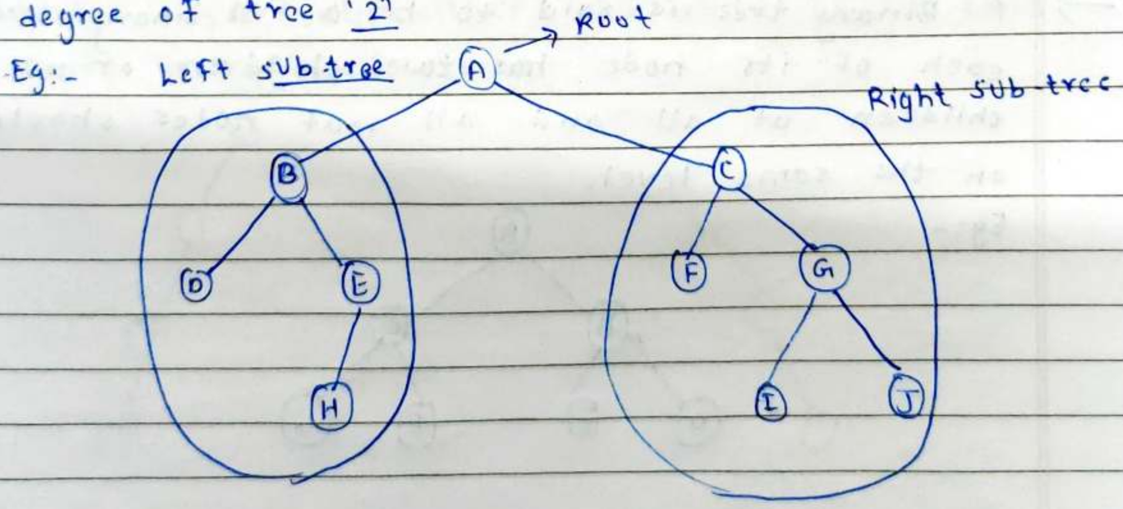
→ ◦ Tree is a sequence of Nodes
◦ There is a starting node known as a root node.
◦ Every node other than root has a parent node
◦



Binary Tree:-

→ A Binary Tree (T) is an m-arry tree with m=2 which has a left subtree and right subtree with degree of tree '2'

Eg:-



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Properties of Binary Tree:-

1. A Tree with n nodes has exactly (n-1) edges or branches.
2. For m nodes at level L, it contains at most '2m' nodes at level L+1.
3. Max. no. of Nodes in binary tree = $2^h - 1$ (h → height of tree)
4. The minimum height of a binary tree with n nodes is $\log_2(n+1) - 1$
5. A Binary tree with n internal nodes has 'n+1' external nodes.

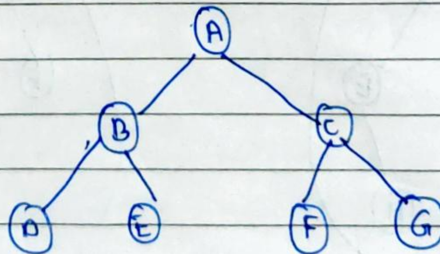
⇒ **TYPES OF Binary Tree:-**

1. Complete Binary Tree
2. Strictly Binary Tree.
3. Full Binary Tree
4. Skewed Binary Tree.

- **Full Binary Tree:-**

→ A Binary tree is said to be a Full binary tree if each of its node has two children or no children at all and all leaf nodes should be on the same level.

Eg:-

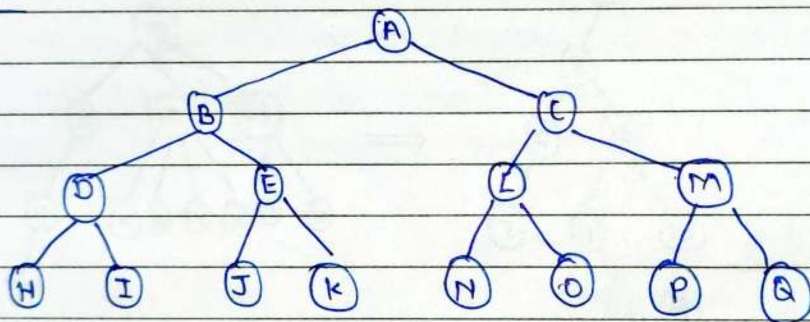


- Complete Binary Tree:-

→ A Complete Binary Tree is a binary tree in which every level, except possibly the last, is completely filled, and all nodes are as far left as possible.

- A complete binary tree of depth d is called strictly binary Tree if all of whose leaves are at level d.
- A complete Binary Tree has 2^d nodes at every depth d and $2^d - 1$ non leaf nodes

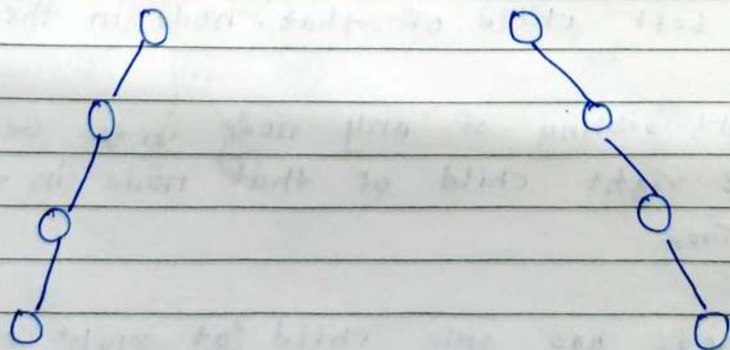
Eg:-



- Skewed Binary Tree:-

→ A skewed binary tree is the one in which all nodes have only either one child or no child.

Types of skewed tree:-



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- **Strictly Binary Tree:-**

→ ① If every non-leaf node in a binary tree has non-empty left and right sub-trees, then such a tree is called a strictly Binary Tree.

② Or, to put in another way, all of the nodes in a strictly binary tree are of degree zero or two, never degree one.

CONVERSION:- General Tree to Binary Tree

- Algorithm:

① The root of the Binary Tree is the root of the General Tree.

② The Left child of a node in the General tree is the Left child of that node in the Binary Tree

③ The Right sibling of any node in the General Tree is the right child of that node in the Binary Tree.

→ For Eg:-

If a node has only child (at right or left), then it will be attached to left in binary Tree (BT)

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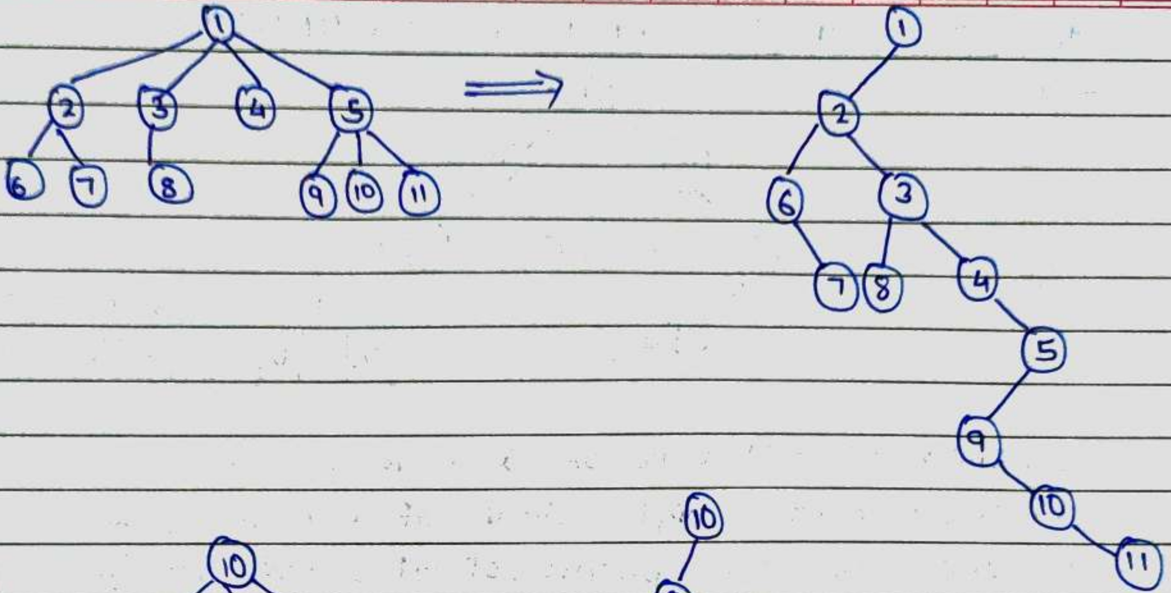
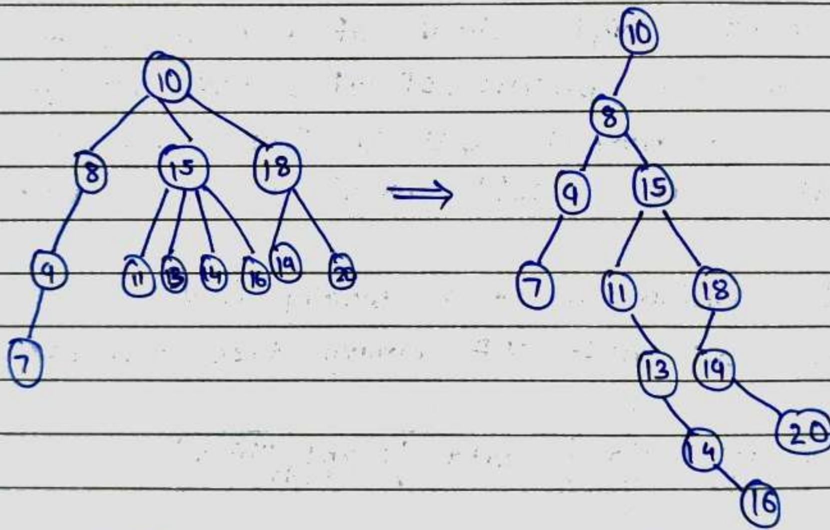
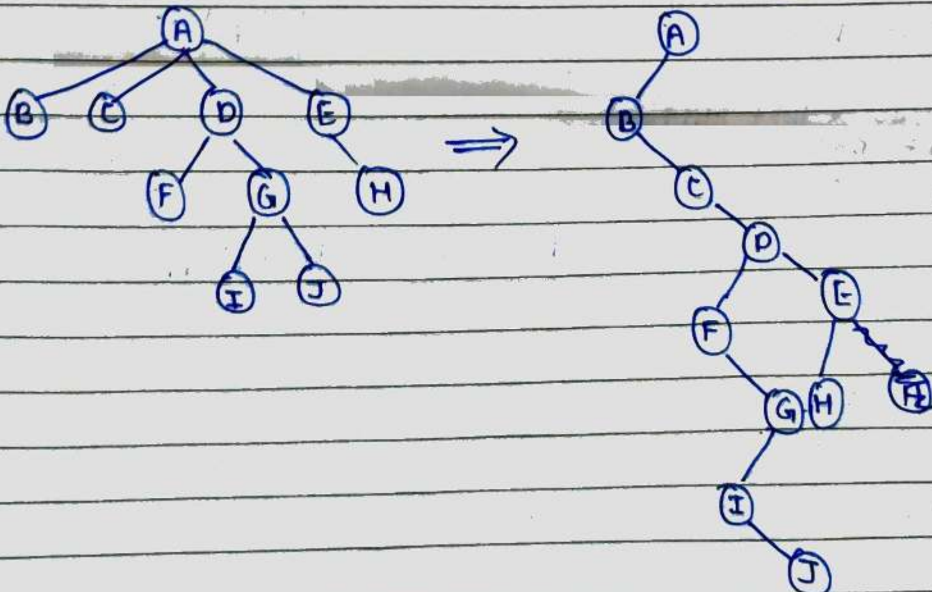
GENERAL TREE BINARY TREE

① 

② 

③ 

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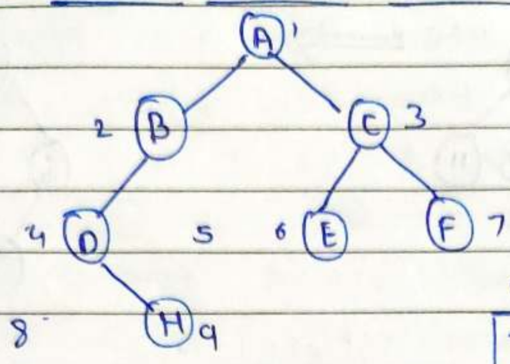
TREE AND IT'S ARRAY REPRESENTATION

→



0	1	2	3	4	5	6	7	8	9
7	A	B	C	D	-	E	F	-	H

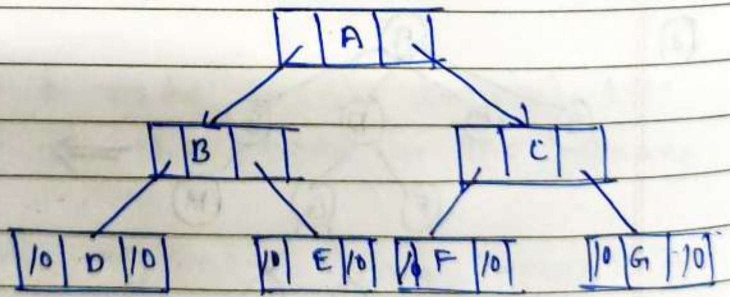
- Index of left child of a node $i = 2i$
- Index of the right child of a node $i = 2i + 1$
- Index of the parent of ~~of~~ a node $i = i/2$.
- Sibling of a node i will be found at the location $i+1$, if i is a left child of its parent.

Linked Organization of a Binary Tree:-

→ To represent node of binary tree, the structure is:

Left Child Address	Data	Right Child Address

```
struct Node
{
  Node *left_child;
  int Data;
  Node *right_child;
};
```



Tree Traversal:-

(1) Pre-Order

Root → Left → Right

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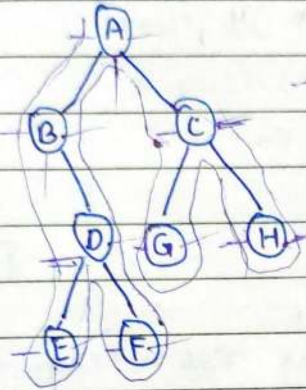
(2) ~~Post~~ In Order

Left → Root → Right.

(3.) Post Order.

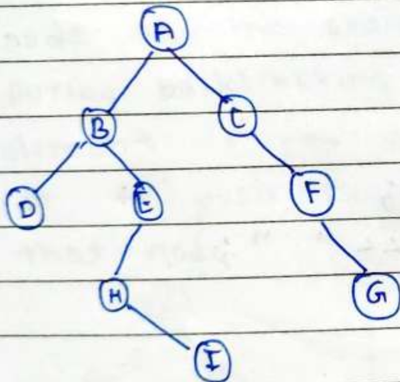
Left → Right → Root.

(1.)



- In Order : BEDFAGCH
- Pre Order : ABDEFCGH
- Post Order : EFDBGHCA

(2)



- In-Order : DBHIEACFG.
- Pre-Order : ABDE HICFG
- PostOrder : DIHEBGFCA

Depth First search.

Recurisive pre-order traversal.

→
```
void preorder (node * root)
{
    if (root ! = NULL)
    {
        cout << root -> data << " ";
        preorder(root -> left);
        preorder(root -> right);
    }
}
```

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Recursive in-order traversal

→
```
void inorder (node *root)
{
    if (root != NULL)
    {
        inorder (root -> left);
        cout << root -> data << " ";
        inorder( root -> right);
    }
}
```

Recursive post-order traversal,

→
```
void postorder( node *root)
{
    if (root != NULL)
    {
        postorder (root -> left);
        postorder (root -> right);
        cout << root -> data << " ";
    }
}
```

Non-recursive pre-order traversal,

→
```
void preorder_nonrec (node *T)
{
    stack s ;
    while (T != NULL)
    {
        cout << T -> data << " ";
        s.push(T);
        T = T -> left;
    }
```

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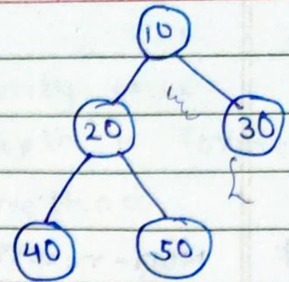
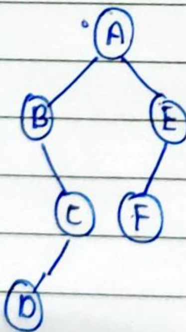
```
while (!s.empty())
{
    T = s.pop();
    T = T -> right;
    while (T! = NULL)
    {
        cout << T -> data << " ";
        s. push (T);
        T = T -> left;
    }
}
}
```



→ <u>For postorder non recursive traversal</u>:-

- While going to the left subtree push all the left side node on the stack with the flag value 0.
- During backtracking travese to the right <u>sub-tree</u>, an element is pop and if the flag value is 0, then it ~~is~~ push flag value with 1. and print data of that node.



B	0
A	0

B	1
A	0

D	0
C	0
B	1
A	0

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Non-recursive Postorder traversal.

```
1.) while (T! = NULL)
    {
        s.push (T) ;  s1. push (NULL)
        T = T → ptr.;
    }
2.) while (! s.empty ())
    {
        T = s.pop();
3.) check the flag is '0'. if(s1. pop( ) = NULL)
    then push that element again with flag 1
        Flag = 1
  { & s.push(T) ; s1. push ((node *) 1)
        T = T → right.
      while ( T! = NULL)
        {   // flag = 0
            s1. push (NULL);
            s.push = (T)
            T = T → 1ptr;
        }
4.) else if when flag = 1
      p.pop() and print the data.
5.) end.
```

Ex.)
①
②

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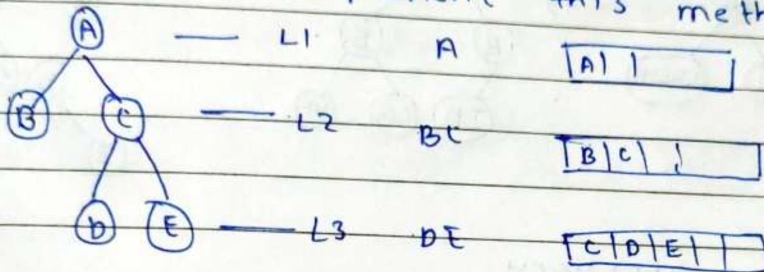
Breadth First Search (BFS)

- It is levelwise traversing the node only once.
- In Breadth First search (BFS), no depth is formed.
- Queue is used to implement this method.

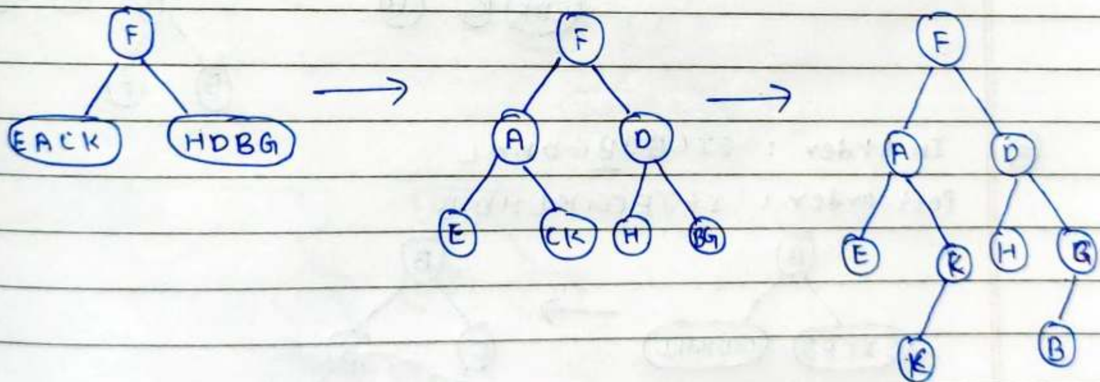


Creation of Binary Tree Inorder, Preorder and Postorder

Ex) With Inorder and preorder.

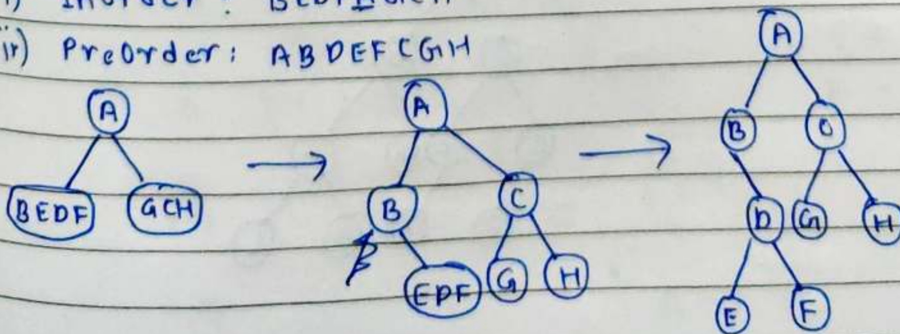
① i) Inorder : EACKFHDBG.

ii) Preorder : FAEKCDHGB.
↳ root



② i) InOrder : BEDFAGCH

ii) PreOrder : ABDEFCGH



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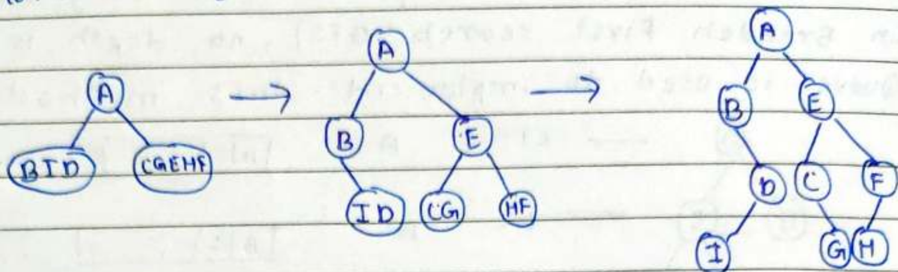
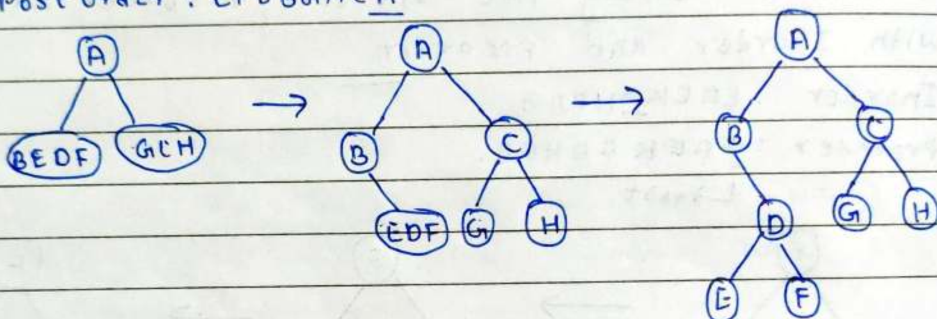
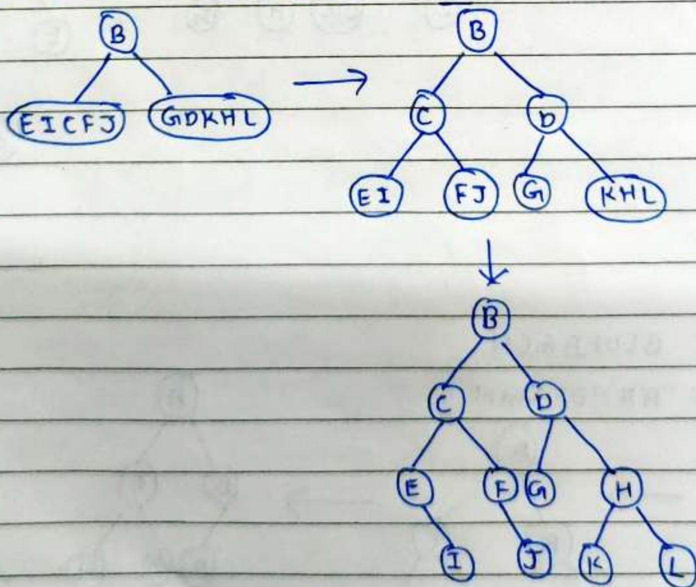
③ In order : BIDACGEHF
Postorder : IDBGCHFEA.



④ In Order : BEDFAGCH
Post Order : EFDBGHCA



⑤ In Order : EICFJBGDKHL
Post order : IEJFCGKLHDB



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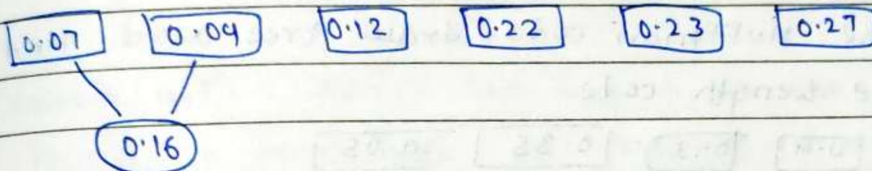
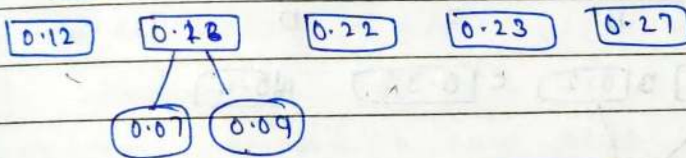
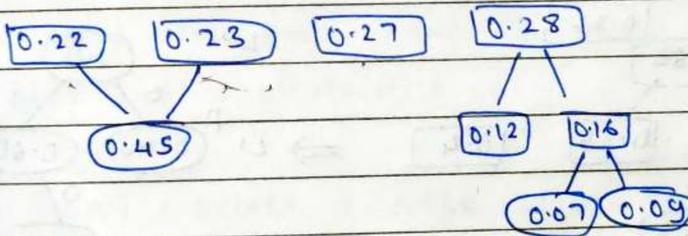
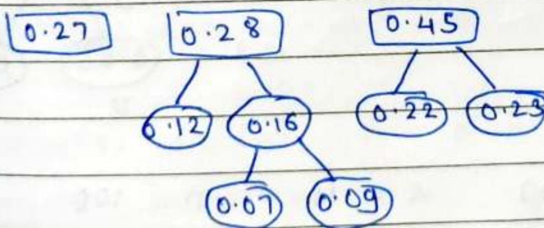
Huffman Coding :-

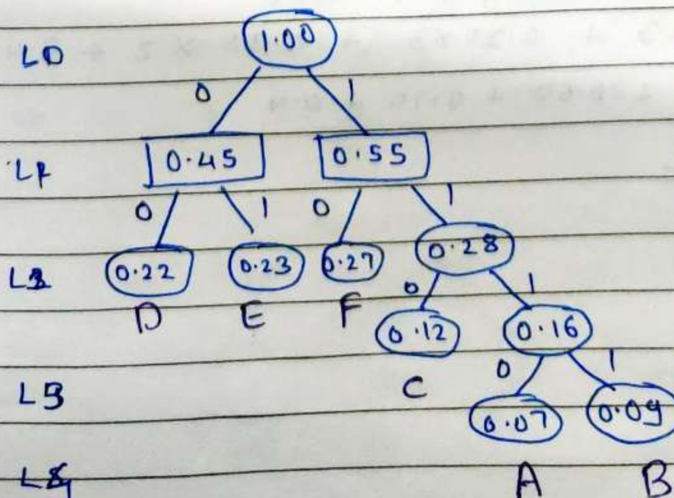
→ Suppose character A, B, C, D, E, F have probabilities 0.07, 0.09, 0.12, 0.22, 0.23 and 0.27 respectively. Find an optimal Huffman code and draw huffamon tree. What is the average code length.











A ≡ 1110

B ≡ 1111

C ≡ 110

D ≡ 00

E ≡ 01

F ≡ 10

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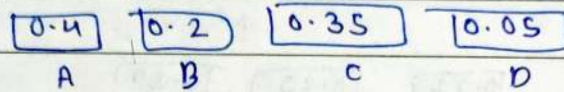
Average length code = $\frac{(0.07\times 4) + (0.09\times 4) + (0.12\times 3) + (0.22\times 2) + (0.23\times 2) + (0.27\times 2)}{6}$

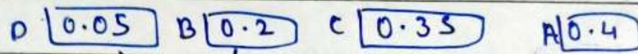
$= 0.028 + 0.36 + 0.36 + 0.44 + 0.46 + 0.54$

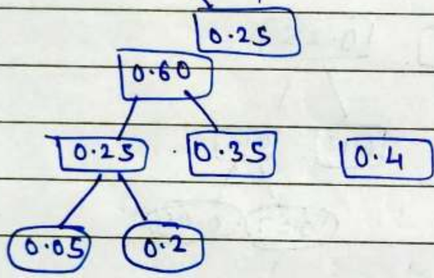
$= 2.44$

Q] Find the Huffman code, draw tree and find the average length code.

→







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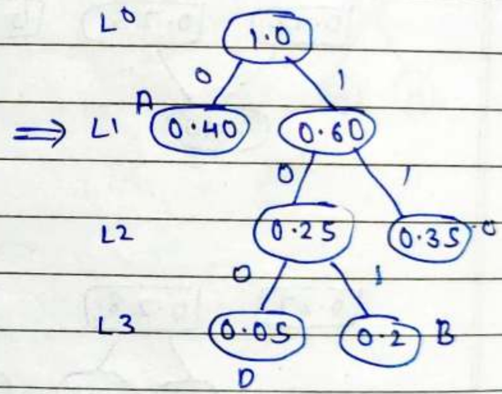

A ≡ 0 B ≡ 101 C ≡ 11 D ≡ 100

Average code length :- ACL

$= 0.05\times 3 + 0.20\times 3 + 0.35\times 2 + 0.4\times 1$

$= 0.15 + 0.60 + 0.70 + 0.4$

$= 1.85$

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Binary Search Tree :-

	Array	LinkedList	BST
search	O(n)	O(n)	O(logn)
Insert	O(1)	O(1)	O(logn)
Delete	O(n)	O(n)	O(log n)

Search (K,T) : Search for key K in the tree T. If K is found in some node of tree then return true otherwise return false

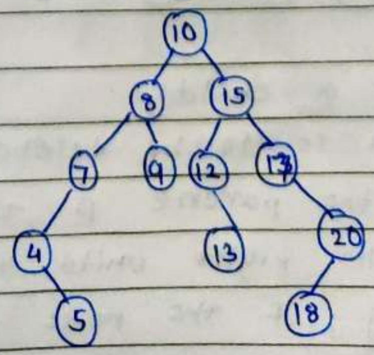
Insert (K,T) : Insert a new node with value K in the info field in the tree T such that the property of BST is maintained.

Delete (K,T) : Delete a node with value K in the info field from the tree T such that the property of BST is maintained.

Find Min(T), Find Max(T) : Find minimum and maximum element from the given non-empty BST.

Construc BST for following Number.

① 10 8 15 12 13 7 9 17 20 18 4 5



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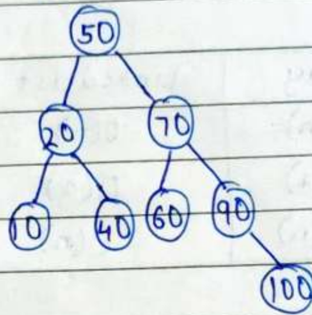
② 50 70 60 20 90 10 40 100.

→

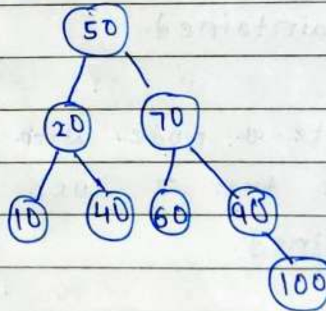


DELETION:- (in BST)

(1.) When node is a Leaf Node

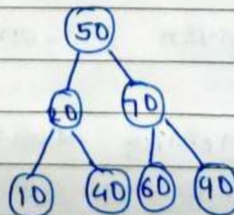
→ If the node to be deleted is leaf node, it can be deleted directly by setting current parent pointer pointing to NULL.

For Eg:-



→ Delete 100.

∴



(2.) When node has a child:-

→ If a node queue is to be deleted and it is right child of the parent p, the only child of q will become the right child of p after deletion of q. Similarly, if the node q is to be deleted and it the left child of parent node p, then only child of q will become the left child of p

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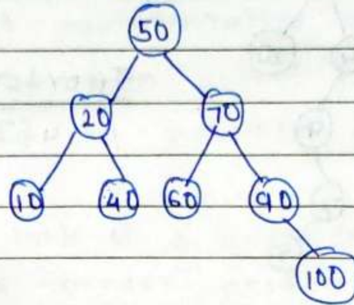
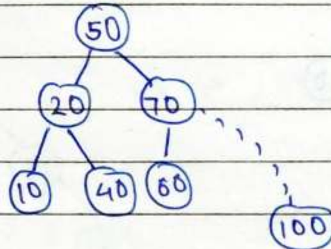
after deletion of 4.



→ Delete 90:-



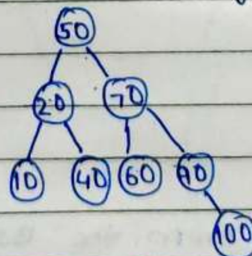
(3.) A node have 2 (two) childs:-

→ The general strategy is to replace the data of this node with smallest data of right side or inorder function.

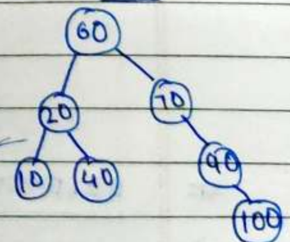
The smallest node in right-sub tree will either be a leaf node or node of degree one,

Then, delete that node using 1

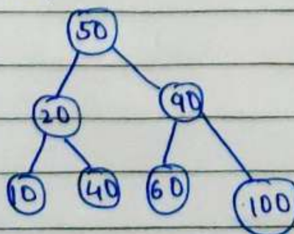
→ Example :-



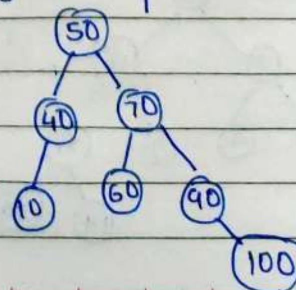
Delete 50



Delete 70:



Delete 20



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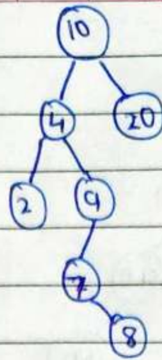
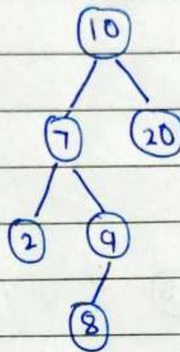
→



Inorder:-
2, 4, 7, 8, 9, 20

Delete 4:-



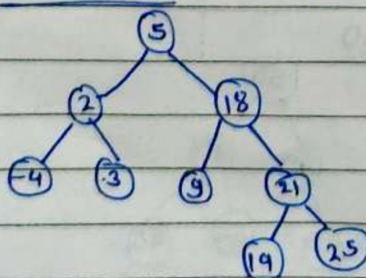
SEARCHING THROUGH the BST.

1. Compare the target value with the element in the root node.

→ • If the target value is equal, the search is successful.
- If the target value is less, search the left subtree.
- If the target value is greater, search the right sub tree.
-

DELETING a node from the BST.

→



Delete 18:-

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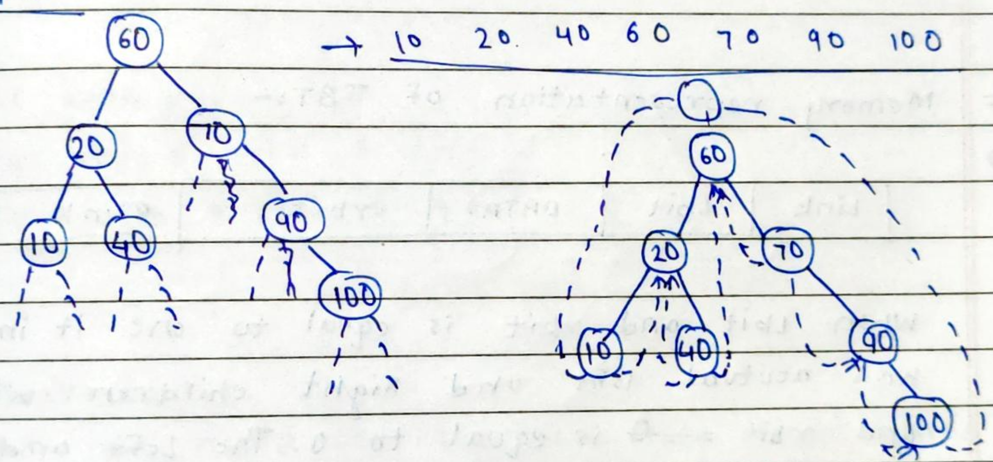
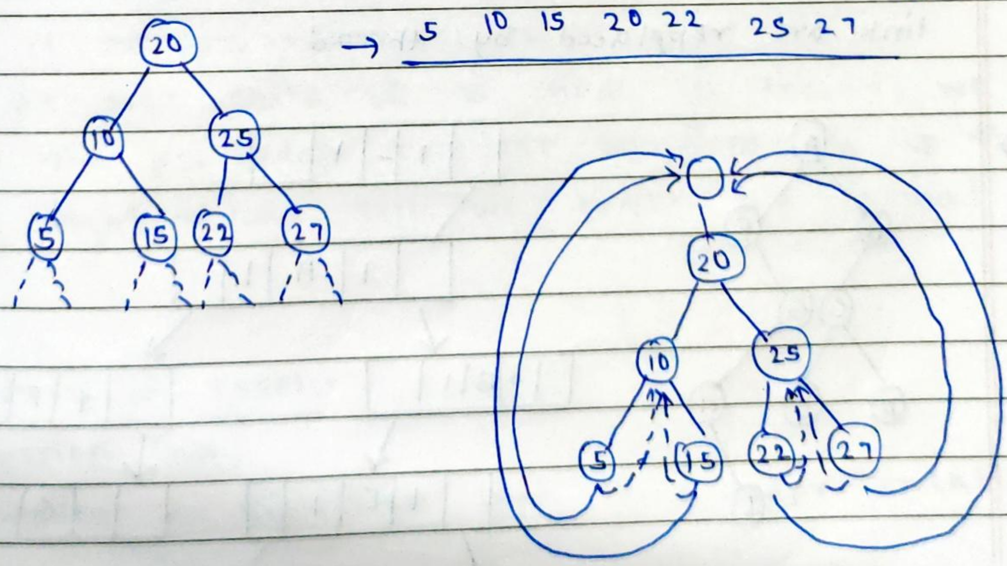
Threaded Binary Tree :- (TBT)

→ In a linked list representation of a binary tree, there are more null links than actual pointers. These null links can be replaced by pointers called threads to other nodes.

- A left null link of a node is replaced with the address of its inorder predecessor and right null link is replaced with address of its inorder successor.

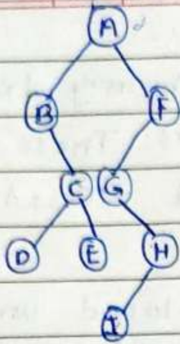
→ Example:-



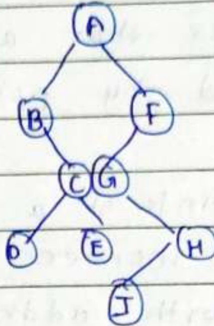


SPPU-SE-COMP-CONTENT – KSKA Git

LRootR

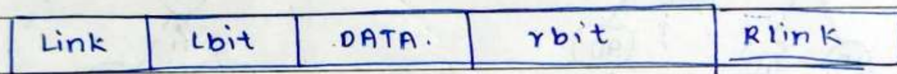


Inorder : BDCEAGIHF



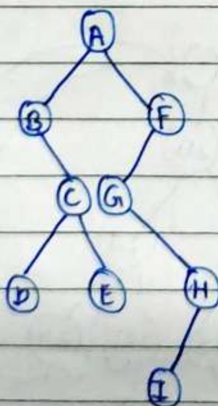
Memory representation of TBT:-

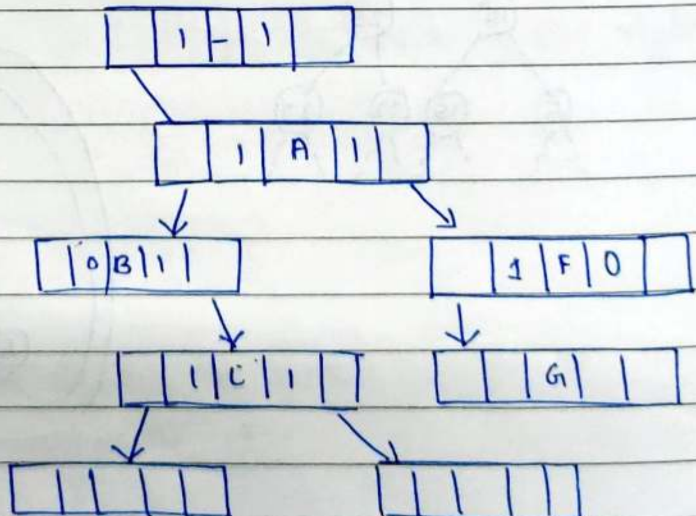
→



When Lbit and rbit is equal to one it indicates it has acutual left and Right children when Lbit and rbit ~~= 0~~ is equal to 0. The Left and right link are repplaced by thread.

→





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Advantage: The time required is reduced due to backtracking using threads.

- No need to use stack. (No requirement)
- We can directly add extra datastructure.

Disadvantage :- ① complex structure. (class TBT node)
② Insertion and Deletion is difficult.

```
class TBT_node
{
    int data;
    TBT_node * lptr, * rptr;
    TBT_node * lbit, * rbit;
}
```

TBT Traversals:-

→ If the left child of the node is actual child then left is the preorder successor.
If the left child of a node is thread, we can locate the pre-order successor by climbing up the tree using right thread till we reach the normal right child.

Application of TREES:-

(1.) Expression Tree

Defination: An Expression tree is a representation

The traversals in the expression tree will prefix, infix, postfix expressions.

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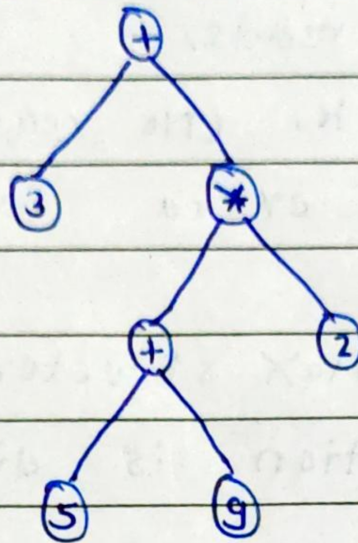
- Expression tree for 3+((5+9)*2) would be:-



(2.) Decision Tree:-