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# Quine-McCluskey Method

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# K-Map Pros and Cons

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- ❑ K-Map is systemic
- ❑ Require the ability to identify and visualize the prime implicants in order to cover all minterms
- ❑ But effective only up to 5-6 input variables!

# Quine-McCluskey Algorithm

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- Tabular Method
  - Compute all prime implicants
  - Find a minimum expression for Boolean functions
- No visualization of prime implicants
- Can be programmed and implemented in a computer

# QM Method Example

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$$F(W, X, Y, Z) = \sum m(0,3,5,6,7,10,12,13) + \sum d(2,9,15)$$

- Step 1 : Divide all the minterms (and don't cares) of a function into groups

**For  
Minterms:**

Minterm ID	W	X	Y	Z
0	0	0	0	0
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
10	1	0	1	0
12	1	1	0	0
13	1	1	0	1

**For don't  
cares:**

Minterm ID	W	X	Y	Z
2	0	0	1	0
9	1	0	0	1
15	1	1	1	1

# QM Method Example












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- Step 1 : Divide all the minterms (and don't cares) of a function into groups

Groups	Minterm ID	W	X	Y	Z	Merge Mark
G0	0	0	0	0	0	
G1	2	0	0	1	0	
G2	3	0	0	1	1	
	5	0	1	0	1	
	6	0	1	1	0	
	9	1	0	0	1	
	10	1	0	1	0	
	12	1	1	0	0	
G3	7	0	1	1	1	
	13	1	1	0	1	
G4	15	1	1	1	1	

# QM Method Example









❖ Step 2: Merge minterms from adjacent groups to form a new implicant table

Groups	Minterm ID	W	X	Y	Z	Merge Mark
G0	0	0	0	0	0	
G1	2	0	0	1	0	
G2	3	0	0	1	1	
	5	0	1	0	1	
	6	0	1	1	0	
	9	1	0	0	1	
	10	1	0	1	0	
G3	12	1	1	0	0	
	7	0	1	1	1	
G4	13	1	1	0	1	
	15	1	1	1	1	

Groups	Minterm ID	W	X	Y	Z
G0'	0, 2	0	0	d	0
G1'	2, 3	0	0	1	d
	2, 6	0	d	1	0
	2, 10	d	0	1	0
G2'	3, 7	0	d	1	1
	5, 7	0	1	d	1
	6, 7	0	1	1	d
	5, 13	d	1	0	1
	9, 13	1	d	0	1
	12, 13	1	1	0	d
G3'	7, 15	d	1	1	1
	13, 15	1	1	d	1

# QM Method Example

□ Step 3: Repeat step 2 until no more merging is possible

Groups	Minterm ID	W	X	Y	Z	Merge Mark
G0'	0, 2	0	0	d	0	
G1'	2, 3	0	0	1	d	
	2, 6	0	d	1	0	
	2, 10	d	0	1	0	
G2'	3, 7	0	d	1	1	
	5, 7	0	1	d	1	
	6, 7	0	1	1	d	
	5, 13	d	1	0	1	
	9, 13	1	d	0	1	
	12, 13	1	1	0	d	
G3'	7, 15	d	1	1	1	
	13, 15	1	1	d	1	

Groups	Minterm ID	W	X	Y	Z
G1''	2, 3, 6, 7	0	d	1	d
	2, 6, 3, 7	0	d	1	d
G2''	5, 7, 13, 15	d	1	d	1
	5, 7, 13, 15	d	1	d	1

# QM Method Example

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- Step 3: Repeat step 2 until no more merging is possible

Groups	Minterm ID	W	X	Y	Z	Merge Mark
G0''	0, 2	0	0	d	0	
G1''	2, 3, 6, 7	0	d	1	d	
	2, 10	d	0	1	0	
G2''	5, 7, 13, 15	d	1	d	1	
	9, 13	1	d	0	1	
	12, 13	1	1	0	d	

**• No more merging possible!**



# QM Method Example

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- Step 4: Put all prime implicants in a cover table (don't cares excluded)

Minterm ID	$\bar{W} \bar{X} \bar{Z}$	$\bar{W}Y$	$\bar{X}Y\bar{Z}$	$XZ$	$WX\bar{Y}$	$W\bar{Y}Z$
0	1					
3		1				
5				1		
6		1				
7		1		1		
10			1			
12					1	
13				1	1	1

Need not include  
don't cares

# QM Method Example

- ❖ Step 5: Identify essential minterms, and hence essential prime implicants

Minterm ID	$\bar{W} \bar{X} \bar{Z}$	$\bar{W}Y$	$\bar{X}Y\bar{Z}$	$XZ$	$WX\bar{Y}$	$W\bar{Y}Z$
0	1					
3		1				
5				1		
6		1				
7		1		1		
10			1			
12					1	
13				1	1	1

E.M.T
  E.P.I

# QM Method Example

❖ Step 6: Add prime implicants to the minimum expression of  $F$  until all minterms of  $F$  are covered

Minterm ID	$\bar{W} \bar{X} \bar{Z}$	$\bar{W}Y$	$\bar{X}Y\bar{Z}$	$XZ$	$WXY\bar{Y}$	$W\bar{Y}Z$
0	1					
3		1				
5				1		
6		1				
7		1		1		
10			1			
12					1	
13				1	1	1

**Already cover all minterms!**

E.M.T     E.P.I

## QM Method Example

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$$F(W, X, Y, Z) = \sum m(0,3,5,6,7,10,12,13) + \sum d(2,9,15)$$

- So after simplification through QM method, a minimum expression for  $F(W, X, Y, Z)$  is:

$$F(W, X, Y, Z) = \overline{W}\overline{X}\overline{Z} + \overline{W}Y + \overline{X}Y\overline{Z} + XZ + WX\overline{Y}$$

# Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

	Step 1			Step 2			Step 3		
2		5							
		9							
3		7							
		11							
		13							
4		15							

List minterms by the number of **1s** it contains.

# Finding Prime Implicants (PIs)

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$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
	5	0101						
	9	1001						
	7	0111						
	11	1011						
	13	1101						
	15	1111						

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# Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
	5	0101	┌	5,7				
	9	1001	2	5,13				
	7	0111	└	9,11				
	11	1011		9,13				
	13	1101	┌	7,15				
			3	11,15				
	15	1111	└	13,15				

Enter combinations of minterms by the number of **1s** it contains.

# Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
⊗	5	0101		5,7	01-1			
⊗	9	1001		5,13	-101			
				9,11	10-1			
⊗	7	0111		9,13	1-01			
⊗	11	1011						
⊗	13	1101		7,15	-111			
				11,15	1-11			
⊗	15	1111		13,15	11-1			

Check off elements used from Step 1.



## Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
⊗	5	0101		5,7	01-1		5,7,13,15	-1-1
⊗	9	1001		5,13	-101		5,13,7,15	-1-1
				9,11	10-1		9,11,13,15	1- -1
⊗	7	0111		9,13	1-01		9,13,11,15	1- -1
⊗	11	1011						
⊗	13	1101		7,15	-111			
				11,15	1-11			
⊗	15	1111		13,15	11-1			

Enter combinations of minterms by the number of **1s** it contains.

# Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
☒	5	0101	☒	5,7	01-1		5,7,13,15	-1-1
☒	9	1001	☒	5,13	-101		5,13,7,15	-1-1
			☒	9,11	10-1		9,11,13,15	1--1
☒	7	0111	☒	9,13	1-01		9,13,11,15	1--1
☒	11	1011						
☒	13	1101	☒	7,15	-111			
			☒	11,15	1-11			
☒	15	1111	☒	13,15	11-1			

The entries left **unchecked** are Prime Implicants.

# Finding Essential Prime Implicants (EPIs)

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	Prime Implicants	Covered Minterms	<u>Minterms</u>					
			5	7	9	11	13	15
	- 1 - 1	5,7,13,15						
	1 - - 1	9,13,11,15						

Enter the Prime Implicants and their minterms.

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## Finding Essential Prime Implicants (EPIs)

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	Prime Implicants	Covered Minterms	<u>Minterms</u>					
			5	7	9	11	13	15
	- 1 - 1	5,7,13,15	X	X			X	X
	1 - - 1	9,13,11,15			X	X	X	X

Enter Xs for the minterms covered.

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# Finding Essential Prime Implicants (EPIs)

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	Prime Implicants	Covered Minterms	<u>Minterms</u>					
			5	7	9	11	13	15
	- 1 - 1	5,7,13,15	X	X			X	X
	1 - - 1	9,13,11,15			X	X	X	X

**Circle** Xs that are in a column singularly.

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# Finding Essential Prime Implicants (EPIs)

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	Prime Implicants	Covered Minterms	<u>Minterms</u>					
			5	7	9	11	13	15
⊗	- 1 - 1	5,7,13,15	X	X			X	X
⊗	1 - - 1	9,13,11,15			X	X	X	X

The **circled Xs** are the **Essential Prime Implicants**, so we check them off.

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# Finding Essential Prime Implicants (EPIs)

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	Prime Implicants	Covered Minterms	Minterms					
			5	7	9	11	13	15
⊗	- 1 - 1	5,7,13,15	X	X			X	X
⊗	1 - - 1	9,13,11,15			X	X	X	X
			⊗	⊗	⊗	⊗	⊗	⊗

We check off the minterms covered by each of the EPIs.

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# Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms					
			5	7	9	11	13	15
⊗	- 1 - 1	5,7,13,15	X	X			X	X
⊗	1 - - 1	9,13,11,15			X	X	X	X
			⊗	⊗	⊗	⊗	⊗	⊗

EPIs:

W	X	Y	Z
-	1	-	1
1	-	-	1

$$F = (X \cdot Z) + (W \cdot Z)$$

$$= (X + W) \cdot Z$$





# Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(2,3,6,7,8,10,11,12,14,15)$$

Step 1			Step 2			Step 3			Step 4		
⊗	2	0010		2,3	001-						
⊗	8	1000		2,6	0-10						
				2,10	-010						
⊗	3	0011		8,10	10-0						
⊗	6	0110		8,12	1-00						
⊗	10	1010									
⊗	12	1100		3,7	0-11						
				3,11	-011						
⊗	7	0111		6,7	011-						
⊗	11	1011		6,14	-110						
⊗	14	1110		10,14	1-10						
				10,11	101-						
⊗	15	1111		12,14	11-0						
				7,15	-111						
				11,15	1-11						
				14,15	111-						

# Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(2,3,6,7,8,10,11,12,14,15)$$

Step 1			Step 2			Step 3			Step 4		
⊗	2	0010	⊗	2,3	001-		2,3,6,7	0-1-			
⊗	8	1000	⊗	2,6	0-10		2,6,3,7	0-1-			
			⊗	2,10	-010		2,3,10,11	-01-			
⊗	3	0011	⊗	8,10	10-0		2,6,10,14	--10			
⊗	6	0110	⊗	8,12	1-00		2,10,3,11	-01-			
⊗	10	1010					2,10,6,14	--10			
⊗	12	1100	⊗	3,7	0-11		8,10,12,14	1--0			
			⊗	3,11	-011		8,12,10,14	1--0			
⊗	7	0111	⊗	6,7	011-						
⊗	11	1011	⊗	6,14	-110		3,7,11,15	--11			
⊗	14	1110	⊗	10,14	1-10		3,11,7,15	--11			
			⊗	10,11	101-		6,7,14,15	-11-			
⊗	15	1111	⊗	12,14	11-0		6,14,7,15	-11-			
							10,14,11,15	1-1-			
			⊗	7,15	-111		10,11,14,15	1-1-			
			⊗	11,15	1-11						
			⊗	14,15	111-						

# Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(2,3,6,7,8,10,11,12,14,15)$$

Step 1			Step 2			Step 3			Step 4		
⊗	2	0010	⊗	2,3	001-	⊗	2,3,6,7	0-1-		2,3,6,7,10,14,11,15	-- 1 -
⊗	8	1000	⊗	2,6	0-10	⊗	2,6,3,7	0-1-		2,3,10,11,6,14,7,15	-- 1 -
			⊗	2,10	-010	⊗	2,3,10,11	-01-		2,6,3,7,10,11,14,15	-- 1 -
⊗	3	0011	⊗	8,10	10-0	⊗	2,6,10,14	-- 10		2,6,10,14,3,7,11,15	-- 1 -
⊗	6	0110	⊗	8,12	1-00	⊗	2,10,3,11	-01-		2,10,3,11,6,7,14,15	-- 1 -
⊗	10	1010				⊗	2,10,6,14	-- 10		2,10,6,14,3,11,7,15	-- 1 -
⊗	12	1100	⊗	3,7	0-11		8,10,12,14	1-- 0			
			⊗	3,11	-011		8,12,10,14	1-- 0			
⊗	7	0111	⊗	6,7	011-						
⊗	11	1011	⊗	6,14	-110	⊗	3,7,11,15	-- 11			
⊗	14	1110	⊗	10,14	1-10	⊗	3,11,7,15	-- 11			
			⊗	10,11	101-	⊗	6,7,14,15	-11-			
⊗	15	1111	⊗	12,14	11-0	⊗	6,14,7,15	-11-			
						⊗	10,14,11,15	1-1-			
			⊗	7,15	-111	⊗	10,11,14,15	1-1-			
			⊗	11,15	1-11						
			⊗	14,15	111-						





# Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms									
			2	3	6	7	8	10	11	12	14	15
	1 - - 0	8,12,10,14					X	X		X	X	
	- - 1 -	2,3,6,7,10,11,14,15	X	X	X	X		X	X		X	X





# Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms									
			2	3	6	7	8	10	11	12	14	15
⊗	1 - - 0	8,12,10,14					X	X		X	X	
⊗	- - 1 -	2,3,6,7,10,11,14,15	X	X	X	X		X	X		X	X
			⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗

# Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms									
			2	3	6	7	8	10	11	12	14	15
⊗	1 - - 0	8,12,10,14					X	X		X	X	
⊗	- - 1 -	2,3,6,7,10,11,14,15	X	X	X	X			X	X		X
			⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗

EPIs:

W	X	Y	Z
1	-	-	0
-	-	1	-

$$F = (W.Z') + Y$$