Mathematical logic

The development of **formal logic** and its implementation is essential in computer science.

It is mainly used for deriving **a conclusion** based on what one already knows.

Logic is the study of **correct reasoning**. It provides rules to determine whether a given **argument** is **valid** or **not**.

Proposition

A proposition is a **statement** that can be either ‘**true**’ or ‘**false**’.

Examples:

1) It rained yesterday.
2) India is a state.
3) Himachal Pradesh is a country.

Proposition

It is possible to determine whether any given sentence is a proposition by **prefixing** it with:

It is true that ……

Or

It is false that ……

and check whether the result makes any grammatical **sense**.

Check whether the following sentences are propositions or not

1) New Delhi is the capital of Sri Lanka.
2) 1 + 1 = 10.
3) Two is less than five.
4) Man will reach Mars by 2050.
5) Close the door.

Proposition

6) This is true.
7) What time is it?
8) 6+5
9) x is greater than 5.
10) What a beautiful day!

Propositional Logic

It is the study of propositions (true or false statements) and ways of **combining them** (logical operators) to get **new propositions**. It is effectively **algebra of propositions**. In this algebra, the **variables** stand for **propositions** and the operators (connectives) are ***and***, ***or***, ***not***, ***implies(if then)***, and ***if and only if***.

Propositional Logic

Connective	Symbol	**Example** (p and q are simple statements)
p and q	**∧**	p **∧** q
p or q	**∨**	p **∨** q
not p	¬ or ~	¬p or ~p
Implies (if then)	→	p → q
if and only if (iff)	↔	p ↔ q

Consider a statement "**Raju will eat fruit-salad if the fruit-salad contains mangoes in it**". The statement is equivalent to the statement "**If the fruit-salad contains mangoes, then Raju will eat it**". The statement is a complex statement constructed from two simple statements say **p** and **q**, where

Propositional Logic

p: Fruit-slalad contains mangoes.
q: Raju will eat fruit-salad containing mangoes.

If p then q, when p and q are propositions can be written as
$p \rightarrow q$.

The above sentence ($p \rightarrow q$) states only that Raju will eat fruit-salad containing mangoes. It **does not**, however, **rule out** the **possibility** that Raju will eat fruit-salad containing apples.

Whenever there is a statement $p \leftrightarrow q$ (**if and only if**),its meaning is different from the previous one. This is equivalent to the statement “If the fruit-salad contains mangoes, then Raju will eat it AND If Raju is eating fruit-salad, then it must be containing mangoes”.

Propositional Logic

The values of the complex statement varies according to the values of its constituent propositions.

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$(p \leftrightarrow q)$ $(q \leftrightarrow p)$
F	F	**F**	**F**	**T**	**T**
F	T	**F**	**T**	**T**	**F**
T	F	**F**	**T**	**F**	**F**
T	T	**T**	**T**	**T**	**T**

Converse and Contrapositive

For a proposition $p \rightarrow q$, the proposition **$q \rightarrow p$** is called its **converse** and the proposition **$\neg q \rightarrow \neg p$** is called **contrapositive**.

Propositional Logic

Truth table for converse and contrapositive

p	q	$p \to q$	$q \to p$	$\neg q \to \neg p$
F	F	T	T	T
F	T	T	**F**	T
T	F	**F**	T	**F**
T	T	T	T	T

Propositional Logic

Q1) Using the following statements

p:Raju is tall.

q:Raju is strong.

What is the symbolic form of the following statement?

“Raju is tall but week.”

Answer: The statement given is equivalent to “Raju is tall and Raju is not strong”. So the corresponding symbolic representation is $(p \wedge \neg q)$.

Propositional Logic

Tautology & Contradiction

A proposition whose truth value is always **true** is called a **tautology** and one whose truth value is always **false** is called a **contradiction**. The negation of a tautology is a contradiction and that of a contradiction is a tautology.

Propositional Logic

2) Let X denotes $(p \lor q) \rightarrow r$ and Y denotes $(p \rightarrow r) \lor (q \rightarrow r)$. Which of the following is a tautology?

a) $X \leftrightarrow Y$ b) $X \rightarrow Y$ c) $Y \rightarrow X$ d) $\neg Y \rightarrow X$

Answer: We need to draw truth tables for all the options given.

p	**q**	**r**	**p ∨ q**	**p → r**	**q→r**	**X**	**Y**	**¬ Y**	**X → Y**	**Y → X**	**¬Y → X**
F	F	F	F	T	T	T	T	F	T	T	T
F	F	T	F	T	T	T	T	F	T	T	T
F	T	T	T	T	T	T	T	F	T	T	T
T	T	T	T	T	T	T	T	F	T	T	T
T	F	F	T	F	F	F	T	F	T	F	T
T	T	F	T	F	F	F	F	T	T	T	F
F	T	F	T	T	T	F	T	F	T	F	T
T	F	T	T	T	T	T	T	F	T	T	T

Propositional Function (Predicates)

Quantifiers

Quantifiers are symbols used with propositional functions. There are two types of quantifiers as shown in the table below.

Name	Symbol	Meaning
Universal Quantifier	$\forall$	“ for all”
Existential Quantifier	$\exists$	“ there exists at least one”

Eg: If N is a set of all positive numbers, then the following statements are true.

$\forall x \in N, (x + 3 > 2)$.

$\exists\, x \in N, (x + 2 < 7)$.