Second Year of Computer Engineering (2019 Course)

(210241): Discrete Mathematics

Teaching Scheme	Credit Scheme	Examination Scheme and Marks
Lecture: 03 Hours/Week	03	Mid_Semester (TH): 30 Marks End_Semester (TH): 70 Marks

Marks weightage per unit for examination						
Unit Number	**I**	**II**	**III**	**IV**	**V**	**VI**
Mid_Semester	15	15	-	-	-	-
End_Semester	-	-	18	17	18	17

Prerequisites: Basic Mathematics

Course Objectives

To introduce several Discrete Mathematical Structures found to be serving as tools even today in the development of theoretical computer science.

1. To introduce students to understand, explain, and apply the foundational mathematical concepts at the core of computer science.
2. To understand use of set, function and relation models to understand practical examples, and interpret the associated operations and terminologies in context.
3. To acquire knowledge of logic and proof techniques to expand mathematical maturity.
4. To learn the fundamental counting principle, permutations, and combinations.
5. To study how to model problem using graph and tree.
6. To learn how abstract algebra is used in coding theory.

Course Outcomes

On completion of the course, learner will be able to –

CO1: Formulate problems precisely, solve the problems, apply formal proof techniques, and explain the reasoning clearly.

CO2: Apply appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations including those in real-life contexts.

CO3: Design and analyze real world engineering problems by applying set theory, propositional logic and to construct proofs using mathematical induction.

CO4: Specify, manipulate and apply equivalence relations; construct and use functions and apply these concepts to solve new problems.

CO5: Calculate numbers of possible outcomes using permutations and combinations; to model and analyze computational processes using combinatorics.

CO6: Model and solve computing problem using tree and graph and solve problems using appropriate algorithms.

CO7: Analyze the properties of binary operations, apply abstract algebra in coding theory and evaluate the algebraic structures.

Learning Resources

❖ **Text Books:**

1. C. L. Liu, “Elements of Discrete Mathematics”‖, TMH, ISBN 10:0-07-066913-9.2.
2. N. Biggs, “Discrete Mathematics”, 3rd Ed, Oxford University Press, ISBN 0 –19-850717–8.

❖ **Reference Books:**

1. Kenneth H. Rosen, “Discrete Mathematics and its Applications”‖, Tata McGraw-Hill, ISBN 978-0-07-288008-3
2. Bernard Kolman, Robert C. Busby and Sharon Ross, “Discrete Mathematical Structures”‖, Prentice-Hall of India /Pearson, ISBN: 0132078457, 9780132078450.
3. Narsingh Deo, “Graph with application to Engineering and Computer Science”, Prentice Hall of India, 1990, 0 –87692 –145 –4.
4. Eric Gossett, “Discrete Mathematical Structures with Proofs”, Wiley India Ltd, ISBN:978-81-265-2758-8.
5. Sriram P.and Steven S., “Computational Discrete Mathematics”, Cambridge University Press, ISBN 13: 978-0-521-73311-3.

Unit III

Counting Principles

Duration: (07 Hours)

Mapping of Course Outcomes: CO2,CO5

Unit-III: Contents

- The Basics of Counting, rule of Sum and Product,
- Permutations and Combinations,
- Binomial Coefficients and Identities,
- Generalized Permutations and Combinations,
- Algorithms for generating Permutations and Combinations.

- **Exemplar/ Case Studies:** Study Sudoku solving algorithms & algorithm for generation of new SUDOKU.
- Study Hank-shake Puzzle and algorithm to solve it.

Introduction

- Suppose that a password on a computer system consists of six, seven, or eight characters. Each of these characters must be a digit or a letter of the alphabet. Each password must contain at least one digit. How many such passwords are there?
- The techniques needed to answer this question and a wide variety of counting problems will be introduced in this section.
- **Counting problems** arise throughout mathematics and computer science.
- For example, we must count the successful outcomes of experiments and all the possible outcomes of these experiments to determine probabilities of discrete events. We need to count the number of operations used by an algorithm to study its time complexity.

Introduction

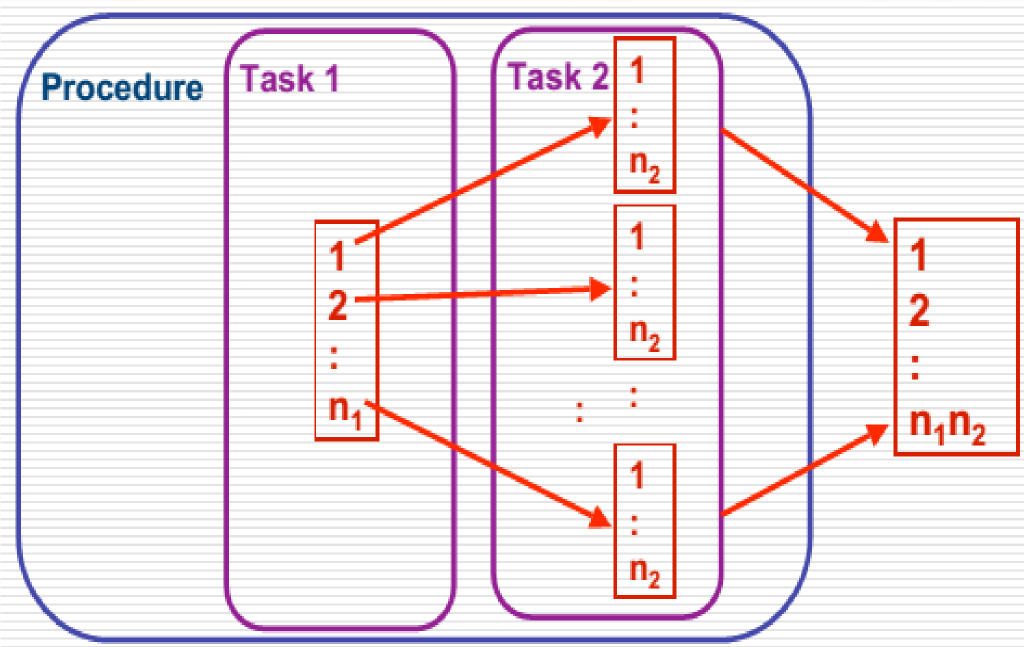
- Counting has many applications in computer science and mathematics.
- For example,
  - Counting the number of operations used by an algorithm to study its time complexity.
  - Counting the successful outcomes of experiments.
  - Counting all the possible outcomes of experiments.

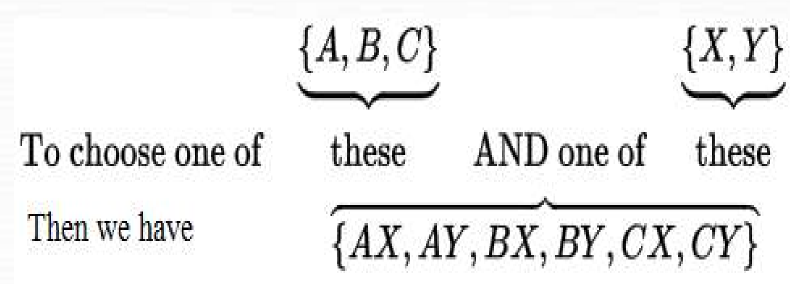
- **Two Basic counting principles:::**
  - 1. The product rule
  - 2. The sum rule

Product Rule

- Suppose that a procedure can be broken down into a sequence of two tasks.
- If there are **n ways** to do the first task and
- for each of these ways of doing the first task, there are **m ways** to do the second task,
- then there are **n*m** ways to do the procedure.

If we wants to make one choice and then another choice, the product rule applies.





Product Rule

- **Example 1:** A new company with just two employees, rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?

  Solution:

  - **Step 1:** Break the procedure into tasks
    - ✓ Task 1: assigning an office to employee 1
    - ✓ Task 2: assigning an office to employee 2
  - **Step 2:** Count different ways of doing each task and then use the product rule
    - ✓ Task 1 can be done in 12 different ways and for each of these ways, Task 2 can be done in 11 different ways.
    - ✓ **By product rule,** There are **12 • 11 = 132** ways to assign offices to two employees.

Product Rule

- **Example 2:** The chairs of an auditorium are to be labeled with a letter and a positive integer not exceeding 100. How many chairs can be labeled differently?

  Solution:

  - **Step 1:** Break the procedure into tasks
    - Task 1: assigning one of the 26 letters
    - Task 2: assigning one of the 100 possible integers
  - **Step 2:** Count different ways of doing each task and then use the product rule
    - Task 1 can be done in 26 different ways and for each of these ways, Task 2 can be done in 100 different ways.
    - **By product rule**, There are **26 • 100 = 2600** ways to assign labels to the chairs.

Product Rule

- **Example 3:** There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?

Solution:

- **Step 1:** Break the procedure into tasks
  - ✓ Task 1: First picking a microcomputer from 32 microcomputers
  - ✓ Task 2: 24 ways to choose the port no matter which microcomputer
- **Step 2:** Count different ways of doing each task and then use the product rule
  - ✓ Task 1 can be done in 32 different ways and for each of these ways, Task 2 can be done in 24 different ways.
  - ✓ **By product rule** shows that there are **32 • 24=768 ports**.

Product Rule

- **Example 4:** An office building contain 27 floors and has 37 offices on each floor. How many offices are there in building?

  Solution: By product rule: There are **27 • 37 = 999** offices.

- **Example 5:** There are 18 mathematics majors and 325 computer science majors at a college. In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

  Solution: By product rule: There are **18 • 325= 5850** ways to pick the two representatives.

Product Rule

- **Example 6:** Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco?

  Solution: By the product rule: flight is determined by choosing an airline for the flight from New York to Denver (which can be done in 6 ways) and then choosing an airline for the flight from Denver to San Francisco (which can be done in 7 ways).

  Therefore there are **6 • 7 = 42** different possibilities for the entire flight.

Extended version of the Product rule

❖ **Extended version of the product rule:**

➢ A procedure can be broken down into a sequence of tasks $T_1, T_2, \ldots, T_m$.

➢ Assume each task T_i $(i=1,2,\ldots,m)$, can be done in n_i different ways, regardless of how the previous tasks were done.

➢ The procedure can be done in $n_1 n_2 \ldots n_m$ different ways.

Extended version of the Product rule

- **Example 7:** How many different bit strings of length seven are there?

Solution:

- **Step 1:** Break the procedure into tasks
  - ✓ Task 1: assigning bit 1 to 0 or 1
  - ✓ Task 2: assigning bit 2 to 0 or 1
  - ✓ …
  - ✓ Task 7: assigning bit 7 to 0 or 1
- **Step 2:** Count different ways of doing each task and then use the product rule
  - ✓ Each task can be done in 2 different ways.
  - ✓ By product rule, there are $\mathbf{2^7 = 128}$ different bit strings of length seven.

Extended version of the Product rule

- **Example 8:** How many different license plates are available if each plate contains a sequence of three letters followed by 3 digits?
  - **Step 1:** Break the procedure into tasks
    - ✓ Task 1: choose letter 1 (26 possible ways)
    - ✓ Task 2: choose letter 2 (26 possible ways)
    - ✓ Task 3: choose letter 3 (26 possible ways)
    - ✓ Task 4: choose digit 1 (10 possible ways)
    - ✓ Task 5: choose digit 2 (10 possible ways)
    - ✓ Task 6: choose digit 3 (10 possible ways)
  - **Step 2:** Count different ways of doing each task and then use the product rule
    - ✓ There are 26 choices for each of the three uppercase English letters and ten choices for each of the three digits. Hence, by the product rule there are a total of **26·26·26·10·10·10= 17,576,000** possible license plates.

Extended version of the Product rule

- **Example 9:** A particular brand of shirt comes in 12 colors, has a male & a female version, & comes in 3 sizes for each sex. How many different types of this shirt are made?

  Solution: By product rule shows that there **12*2*3=72** different types of shirt.

- **Example 10:** How many functions are there from a set with m elements to a set with n elements?
  - Solution: Step 1: Break the procedure into tasks:
  - ✓ Task: Assign an element in codomain to element a_i in domain. For all i=1,2,…,m.
  - ✓ i.e for each of the m elements in the domain, there are n choices from the codomain.
  - Step 2: Count different ways of doing each task and then use the product rule there are $\mathbf{n \cdot n \cdot n \cdots n = n^m}$ different functions.

Extended version of the Product rule

- ❖ **Example 11**: The format of telephone numbers in North America is specified by a numbering plan called as North American numbering plan (NANP) specifies the format of telephone numbers in the U.S., Canada, and many other parts of North America. Let X denote a digit between 0 and 9. Let N denote a digit between 2 and 9. Let Y denote a digit between 0 and 1. In the old plan, the format of telephone numbers is NYX-NNX-XXXX. In the new plan, the format of telephone numbers is NXX-NXX-XXXX. How many North American telephone numbers are possible under the old plan and under the new plan?

Extended version of the Product rule

❖ **Solution of Example 11:** By the product rule, there are 8·2·10=160 area codes with format NYX and 8·10·10=800 area codes with format NXX.

Similarly, by the product rule, there are 8·8·10=640 office codes with format NNX.

The product rule also shows that there are 10·10·10·10=10,000 station codes with format XXXX.

Consequently, applying the product rule again, it follows that under the old plan there are **160·640·10,000=1,024,000,000** different numbers available in North America.

Under the new plan, there are **800·800·10,000=6,400,000,000** different numbers available.

Extended version of the Product rule

❖ **Example 12:** In how many ways can we select 3 students from a group of 5 students to stand in line for a picture? In how many ways can we arrange all 5 of these students in a line for a picture?

Solution:

Note that the order in which we select the students matters. There are five ways to select the first student to stand at the start of the line. Once this student has been selected, there are four ways to select the second student in the line. After the first and second students have been selected, there are three ways to select the third student in the line.

Extended version of the Product rule

❖ **Solution of Example 12 continued**:

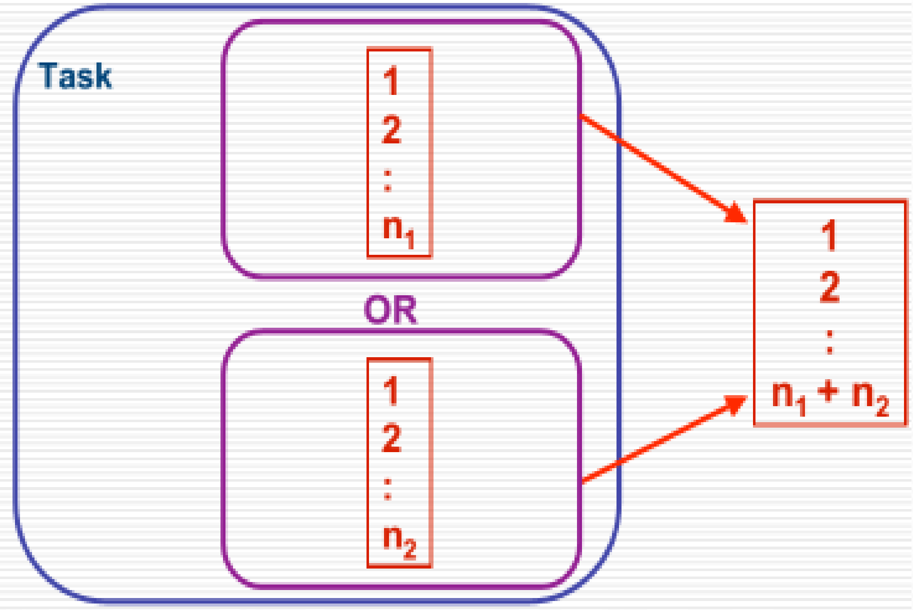
➢ By the product rule, there are 5•4•3 = 60 ways to select three students from a group of five students to stand in line for a picture.

➢ To arrange all five students in a line for a picture, we select the first student in five ways, the second in four ways, the third in three ways, the fourth in two ways, and the fifth in one way.

➢ Consequently, there are 5•4•3•2•1=120 ways to arrange all five students in a line for a picture.

Sum Rule

- If A and B are disjoint, i.e., if $A \cap B = \emptyset$, then $|A \cup B| = |A| + |B|$.
- If there are **n** ways for task one, and
- **m** ways for another task and
- the two tasks cannot be done at the same time, then
- there are **n + m** ways to choose one of these tasks.



If we want to make one choice or another choice, the sum rule applies

Sum Rule

- **Example 13:** Suppose that either a member of the mathematics faculty or a student who is mathematics major is chosen as a representative to a university committee. How many different choices are there for this representative if there are 37 members of the mathematics faculty and 83 mathematics majors and no one is both a faculty member and a student?

  Solution: There are 37 ways to choose a member of the mathematics faculty and there are 83 ways to choose a student who is mathematics major.

  Choosing a member of the mathematics faculty is never the same as choosing a student who is mathematics major because no one is both a faculty member and a student.

  By the sum rule it follows that there are **37 + 83 = 120** possible ways to pick this representative.

Extended version of the Sum rule

❖ **Extended version of the Sum rule:**

❖ Suppose a task can be done in one of $\mathbf{n_1}$ ways, in one of $\mathbf{n_2}$ ways, …, or in one of $\mathbf{n_m}$ ways.

❖ Assume none of the set of $\mathbf{n_i}$ ways of doing the task is the same as any of the set of $\mathbf{n_j}$ ways, for all pairs i and j with $1 \leq i < j \leq m$.

❖ The task can be done in $\mathbf{n_1 + n_2 + \ldots + n_m}$ different ways,

Extended version of the Sum rule

- **Example 14:** A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

  Solution: The student can choose a project by selecting a project from the first list, the second list, or the third list. Because no project is on more than one list, by the sum rule there are **23 + 15 + 19 = 57** ways to choose a project.

- **Example 15**: A university consists only of 2 faculties, namely Science and Engineering. On Tuesday at 11am there are 10 exams that can take place from the faculty of Science. At the same time there are 5 exams that can take place from the faculty of Engineering. How many possible exams can take place?

  Solution: For University there are **10 + 5 = 15** exams that can take place.

Combination of Both Rules

- **Example 16:** In a version of the computer language BASIC, the name of a variable is a string of one or two alphanumeric characters, where uppercase and lowercase letters are not distinguished. (An alphanumeric character is either one of the 26 English letters or one of the 10 digits.) Moreover, a variable name must begin with a letter and must be different from the five strings of two characters that are reserved for programming use. How many different variable names are there in this version of BASIC?

  Solution:

  - Task 1: choose a one character long variable name.
  - Task 2: choose a two characters long variable name.

Combination of Both Rules

- **Solution of Example 16 continued :**
  - Task 1: choose a one character long variable name.
  - Task 2: choose a two characters long variable name.
- ✓ Since the first character of the variable name must be a letter, task 1 can be done in 26 different ways.
- ✓ To choose a two characters long variable name, the first character can be a letter and the second can be a letter or a digit.
- ✓ A two character long variable name must also be different from the five reserved strings.
- ✓ By the product rule, task 2 can be done **in 26 . 36 - 5 = 931** different ways.
- ✓ By the sum rule, there are **931 + 26= 957** different variable names.

Combination of Both Rules

❖ **Example 17:** In a computer system, each user has a password.

a) The password must be six to eight characters long.

b) Each character is an uppercase letter or a digit.

c) Each password must contain at least one digit.

How many possible passwords are there?

Solution:

✓ Task 1: choose a six characters long password.

✓ Task 2: choose a seven characters long password.

✓ Task 3: choose a eight characters long password.

Combination of Both Rules

❖ **Solution of Example 17 continued :**

- **The number of task 1:** the number of passwords of six characters where each character can be an uppercase letter or a digit is 36^6 excluding the number of six characters long passwords with no digits which is 26^6.
- Task 1 can be done in $\mathbf{36^6 - 26^6 = 1,867,866,560}$ ways.

- **The number of task 2:** the number passwords of seven characters where each character can be an uppercase letter or a digit is 36^7 excluding the number of seven characters long passwords with no digits which is 26^7.
- Task 2 can be done in $\mathbf{36^7 - 26^7 = 70,332,353,920}$ ways.

Combination of Both Rules

❖ **Solution of Example 17 continued :**

- **The number of task 3:** the number passwords of eight characters where each character can be an uppercase letter or a digit is 36^8 excluding the number of eight characters long passwords with no digits which is 26^8 .
- Task 3 can be done in $\mathbf{36^8 - 26^8 = 2{,}612{,}282{,}842{,}880}$ ways.

- **By the Sum rule**,

✓ the number of possible passwords are the number of task 1 + the number of task 2 + the number of task 3 = **2,684,483,063,360.**

The Subtraction Rule

❖ **(Inclusion–Exclusion for Two Sets):**

➢ If a task can be done in either **n1 ways** or **n2 ways**, then the number of ways to do the task is **n1 + n2** minus the number of ways to do the task that are common to the two different ways.

➢ The subtraction rule is also known as the **principle of inclusion–exclusion**.

The Subtraction Rule

- **Example 18:** How many bit strings of length eight either start with a 1 bit or end with two bits 00?

Solution: Task 1: choose a bit strings of length eight starts with 1.

Task 2: choose a bit strings of length eight ends with 00.

- ✓ Since the first bit must be 1 in task 1, by the product rule, the number of task 1 is $\mathbf{2^7}$.

1							

- ✓ Since the last two bits must be 00 in task 2, by the product rule, the number of task 2 is $\mathbf{2^6}$.

						0	**0**

- ✓ The strings of length eight that start with 1 and end 00 are common in task 1 and task 2 and the number of them are $\mathbf{2^5}$.

1						**0**	**0**

- ✓ **By inclusion-exclusion principles**, the number of such strings is $\mathbf{2^7 + 2^6 - 2^5 = 160}$.

The Subtraction Rule

- **Example 19:** $|A \cup B|$?

  Solution:

  - Task 1: choose an element from A.
  - Task 2: choose an element from B.
    - ✓ Since A has $|A|$ elements, the number of task 1 is $|A|$.
    - ✓ Since B has $|B|$ elements, the number of task 2 is $|B|$.
    - ✓ The elements that are in both A and B are common in task 1 and task 2 and the number of them are $|A \cap B|$.
    - ✓ **By inclusion-exclusion principles**, the number of task 1 and task 2

$$|A \cup B| = |A| + |B| - |A \cap B|$$

The Subtraction Rule

- **Example 20:** A computer company receives 350 applications for a job. Suppose that 220 of them majored in computer science, 147 of them majored in business and 51 of them majored both in computer science and business. How many of these applicants majored in neither computer science nor business?

  Solution:

  First find the number of applicants that are majored in computer science or business, then subtract it from the total number of applicants

  Task 1: choose an applicant majored in computer science

  Task 2: choose an applicant majored in business.

The Subtraction Rule

❖ **Solution of Example 20 continued :**

- The number of task 1 is 220.
- The number of task 2 is 147.
- The applicants that are majored both in computer science and business are common in task 1 and task 2 and the number of them are 51.
- **By inclusion-exclusion principles**, the number of task 1 and task 2 is 220 + 147 - 51 = 316.
- So, the number of applicants majored in neither computer science and business is 350 – 316 = 34

Examples

❖ **Example 21:** There are 18 mathematics majors and 325 computer science majors at a college.

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

Solution: a)

- There are 18 ways to choose the mathematics major.
- There are 325 ways to choose the computer science major.
- By the product rule, **18*325 = 5850 ways** to pick the two representatives.

Examples

❖ **Example 21:** There are 18 mathematics majors and 325 computer science majors at a college.

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

Solution: b)

➢ There are 18 ways to choose a mathematics major, and there are 325 ways to choose a computer science major.

➢ By sum rule, there are **18 + 325 = 343** ways to pick the representative.

Examples

- **Example 22:** A multiple-choice test contains 10 questions. There are four possible answers for each question.
  a) In how many ways can a student answer the questions on the test if the student answers every question?
  b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

Solution: a) The product rule applies, since the student will perform each of 10 tasks, one after the other. There are 4 ways to do each task.

Therefore there are $4 \cdot 4 \cdots 4 = 4^{10} = 1{,}048{,}576$ ways to answer the questions on the test.

b) This is identical to part (a), except that now there are 5 ways to answer each question- give any of the 4 answers or give no answer at all.

Therefore there are $5^{10} = 9{,}765{,}625$ ways to answer the questions on the test.

Examples-(Homework)

❖ **Example 23:** How many strings of eight English letters are there

a) that contain no vowels, if letters can be repeated?

b) that contain no vowels, if letters cannot be repeated?

c) that start with a vowel, if letters can be repeated?

d) that start with a vowel, if letters cannot be repeated?

e) that contain at least one vowel, if letters can be repeated?

f) that contain exactly one vowel, if letters can be repeated?

g) that start with X and contain at least one vowel, if letters can be repeated?

h) that start and end with X and contain at least one vowel, if letters can be repeated?

Examples-(Homework)

❖ **Example 24:** How many strings of eight uppercase English letters are there

a) if letters can be repeated?

b) if no letter can be repeated?

c) that start with X, if letters can be repeated?

d) that start with X, if no letter can be repeated?

e) that start and end with X, if letters can be repeated?

f) that start with the letters BO (in that order), if letters can be repeated?

g) that start and end with the letters BO (in that order), if letters can be repeated?

h) that start or end with the letters BO (in that order), if letters can be repeated?

Permutations

- The act of arranging all the members of a set into some sequence or order T.

  OR

- An arrangement in sequence of elements of a set is called a **permutation of the element.**
- In a permutation, the order that we arrange the objects in is important.

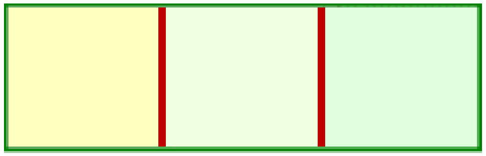
- Depending on the nature of arrangement, there are 4 type of permutation.
  A. Permutation with all object are distinct:
  B. Permutations with Repetition:
  C. Permutations with Indistinguishable Objects:
  D. Circular Permutation:

Permutations

❖ **Example 25:** Consider arranging 3 letters: A, B, C. How many ways can this be done?

Solution: The possible permutations are:

ABC, ACB, BAC, BCA, CAB, CBA.



- Hence, there are six distinct arrangements.
- Another way of looking at this question is by drawing 3 boxes.
- Any one of the A, B, C goes into the first box (3 ways to do this), and then the remaining one of the two letters goes into the second box (2 ways to do this), and the last remaining letter goes into the third box (only one way left to do this).
- Hence, **Total no of ways = $3 \times 2 \times 1 = 6$.**

Permutations

❖ **Theorem I:** If n is a positive integer and r is an integer with $1 \le r \le n$, then there are **$P(n, r) = n(n-1)(n-2)\cdots(n-r+1)$** r-permutations of a set with n distinct elements.

- **Proof:** The first element of the permutation can be chosen in **n ways** because there are n elements in the set.
- There are **(n−1) ways** to choose the second element of the permutation, because there are (n−1) elements left in the set after using the element picked for the first position.
- Similarly, there are **(n−2) ways** to choose the third element, and so on, until there are exactly $n - (r - 1) = n - r + 1$ ways to choose the r^{th} element.
- Consequently, by the product rule, there are **$n(n-1)(n-2)\cdots(n-r+1)$.**

Type of Permutations

❖ Type I: Permutation with all object are distinct:

➢ A permutations of ‘n’ different objects taken ‘r’ at a time is an arrangement of r objects out of n objects where $0 \leq r \leq n$.

➢ It is called r- permutations of n elements and its denoted by P(n,r) or ${}^{n}P_{r}$.

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

➢ **Example 26:** In how many ways can a supermarket manager display 5 brands of biscuits in 3 spaces on a shelf?

Solution: This is asking for the number of permutations, since we don't want repetitions. The number of ways is:

$$ {}^{n}P_{r} = n!/(n-r)! = 5!/(5-3)! = 5!/2! = 60 $$

Type of Permutations

❖ **Theorem II:** Number of permutations of 'n' different things taken 'r' at a time is given by:-

$$^{n}P_{r} = \frac{n!}{(n-r)!} \qquad \text{(n and r are integers with } 0 \le r \le n\text{)}$$

Proof:

- Say we have 'n' different things a1, a2……, an.
- Clearly the first place can be filled up in **'n'** ways. Number of things left after filling-up the first place = (n-1).
- So the second-place can be filled-up in (**n-1**) ways. Now number of things left after filling-up the first and second places = (n – 2).

Type of Permutations

- Now the third place can be filled-up in **(n-2)** ways.
- Thus Number of ways of filling-up r^{th} place = **n – (r - 1) = n - r + 1**
- By product rule of counting, total no. of ways of filling up, first, second r^{th} place together :- **n (n-1) (n-2) ------------ (n - r + 1).**
- Hence: $^{n}P_{r}$ **= n (n-1)(n-2) ---------------(n - r + 1)**
- When n and r are integers with $1 \leq r \leq n$, by Theorem 1 we have

$$P(n, r) = n(n-1)(n-2) \cdots (n-r+1) = n!/(n-r)!$$

- Because $n!/(n-0)! = n!/n! = 1$ whenever n is a nonnegative integer.

$$^{n}P_{r} = \frac{n!}{(n-r)!} \qquad 0 \leq r \leq n$$

Hence proved

Type of Permutations

❖ **Theorem III:** Number of permutations of 'n' different things taken all at a time is given by:- $^nP_n = n!$

Proof:

We have $^nP_r = n!/(n-r)!$

Putting r = n, we have,

$^nP_n = n! / (n-n)!$

$= n!/0!$ Since 0!= 1 we have

$$^nP_n = n!$$

Hence proved

Example :Type of Permutations

❖ **Example 27:** How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

Solution: Because it matters which person wins which prize, the number of ways to pick the three prize winners is the number of ordered selections of three elements from a set of 100 elements, that is, the number of 3-permutations of a set of 100 elements.

Consequently, the answer is **$P(100,3)=100\cdot 99\cdot 98=970{,}200$.**

Example :Type of Permutations

- **Example 28:** How many different number-plates for cars can be made if each number-plate contains four of the digits 0 to 9 followed by a letter A to Z, assuming that no repetition of digits is allowed?

  Solution:

  There are 10 possible digits (0, 1, 2… 9) and we need to take them 4 at a time. There are 26 letters in the alphabet.

$$^{n}P_{r} = n!/(n-r)!$$

$$^{10}P_{4} * 26 = [10!/(10-4)!\ ] * 26 = 131040.$$

Example :Type of Permutations

❖ **Example 29:** How many four digit number can be formed out of digits 1 to 9 if a) No repetition is permitted & b) How many of these will be greater than 3000.

Solution: a) There are 9 possible digits (1, 2,…, 9) and we need to take them 4 at a time. $^{n}P_{r} = n!/(n-r)!$

$$^{9}P_{4} = [9!/(9-4)!] = 9!/5! = 3024$$

b) There is restriction that the four digit numbers so formed must be greater than 3000.

Therefore 1000th position can be filled with numbers 3,4,5,6,7,8,9 i.e. by 7 different ways, 100th position can be filled with 8 different ways, 10th place can be filled with 7 different ways, Unit place can be filled with 6 different ways. Thus, 7 * 8 * 7 * 6 = 2352 ways.

Example :Type of Permutations

❖ **Example 30:** How many permutations of the letters ABCDEFG contain

a) the string BCD?

b) the string CFGA?

c) the strings BA and GF?

d) the strings ABC and DE?

e) the strings ABC and CDE?

f) the strings CBA and BED?

Solution:

a) If BCD is to be a substring, then we can think of that block of letters as one super letter, and the problem is to count permutations of five items-the letters A, E, F, and G, and the super letter BCD. Therefore the answer is $P(5, 5) = 5! = 120$.

b) Reasoning as in part (a), we see that the answer is $P(4,4) = 4! = 24$.

Example :Type of Permutations

❖ **Example 30:** How many permutations of the letters ABCDEFG contain

a) the string BCD?

b) the string CFGA?

c) the strings BA and GF?

d) the strings ABC and DE?

e) the strings ABC and CDE?

f) the strings CBA and BED?

Solution:

c) As in part (a), we glue BA into one item and glue GF into one item. Therefore we need to permute five items, and there are P(5, 5) = 5! = 120 ways to do it.

d) This is similar to part (c). Glue ABC into one item and glue DE into one item, producing four items, so the answer is P(4,4) = 4! = 24.

Example :Type of Permutations

❖ **Example 30:** How many permutations of the letters ABCDEFG contain

a) the string BCD?

b) the string CFGA?

c) the strings BA and GF?

d) the strings ABC and DE?

e) the strings ABC and CDE?

f) the strings CBA and BED?

Solution:

e) If both ABC and CDE are substrings, then ABCDE has to be a substring. So we are really just permuting three items: ABCDE, F, and G. Therefore the answer is $P(3,3) = 3!$

f) There are no permutations with both of these substrings, since B cannot be followed by both A and E at the same time.

Type of Permutations

❖ **Type II: Permutations with Repetition:**

➢ The number of r-permutations of a set of n objects with repetition allowed is $\mathbf{n^r}$. i.e.

➢ The numbers of different permutations of **n** distinct objects taken **r** at a time when every object is allowed to repeat any number of times is given by $\mathbf{n^r}$.

$$P(n,r) = n^r$$

➢ **Example 31:** How many three digit numbers can be formed with the digits: 1, 2, 3, 4, 5?

Solution: n = 5 r = 3. The order of the elements does matter. The elements are repeated. $\mathbf{P(n,r) = n^r}$ **i.e.** $\mathbf{P(5,3) = 5^3 = 125}$

Type of Permutations

- **Example 32:** How many 3 digit numbers can be formed with the digits: 0, 1, 2, 3, 4, 5?

  Solution: n = 6   r = 3. The numbers must be separated into two blocks:

  The first set, of one number, can occupy only one of 5 digits because a number does not begin with zero (except for license plates and other special cases).   n = 5   r = 1

  The second block, of two numbers, can occupy any digit.   n = 6   r = 2

  $P(n,r)= n^r$ i.e. $P(5,1).\ P(6,2) = 5^1 . 6^2 = 180$

Type of Permutations

❖ **Type III: Permutations with Indistinguishable Objects:**

➢ The number of different permutations of n objects of which r_1 are of one kind, r_2 are of a second kind, r_3 are of third kind……. r_k are of k^{th} kind and n = r1 + r2 + ... + rk is given by:

$$P(n,r) = \frac{n!}{r1!r2!r3!.....rk!}$$

➢ **Example 33:** How many ordered arrangements are there of the letters in the word PHILIPPINES?

Solution: The number of ordered arrangements of the letters in the word PHILIPPINES is: r1= P =3, r2=H=1, r3=I=3, r4=L=1, r5=N=1, r6=E=1,r7=S=1

$$\frac{11!}{3!\,1!\,3!\,1!\,1!\,1!\,1!} = 1{,}108{,}800$$

Type of Permutations

- **Example 34:** The signal mast of a ship can raise nine flags at one time (three red, two blue and four green). How many different signals can be communicated by the placement of these nine flags?

Solution: n = 9 r1 = 3, r2 = 2, r3 = 4, r1 + r2 + r3 = 9.

$$\frac{9!}{3!\,2!\,4!} = 1260$$

- **Example 35:** How many nine-digit numbers can be formed with the numbers 2, 2, 2, 3, 3, 3, 3, 4, 4?

Solution: n = 9 r1 = 3, r2 = 4, r3 = 2, r1 + r2 + r3 = 9.

The order of the elements does matter. The elements are repeated.

$$\frac{9!}{3!\,4!\,2!} = 1260$$

Type of Permutations

- **Example 36:** Fifteen (15) animals are available to use in a study to compare three (3) different diets. Each of the diets (let's say, A, B, C) is to be used on five randomly selected animals. In how many ways can the diets be assigned to the animals?

Solution: Well, one possible assignment of the diets to the animals would be for the first five animals to be placed on diet A, the second five animals to be placed on diet B, and the last 5 animals to be placed on diet C. That is:

A A A A A B B B B B C C C C C

Another possible assignment might look like this:

A B C A B C A B C A B C A B C

Type of Permutations

Solution: Well, one possible assignment of the diets to the animals would be for the first five animals to be placed on diet A, the second five animals to be placed on diet B, and the last 5 animals to be placed on diet C. That is:

A A A A A B B B B B C C C C C

Another possible assignment might look like this:

A B C A B C A B C A B C A B C

Upon studying these possible assignments, we see that we need to count the number of distinguishable permutations of 15 objects of which 5 are of type A, 5 are of type B, and 5 are of type C. Using the formula, we see that there are:

$$\frac{15!}{5!\,5!\,5!} = 75675$$

ways in which 15 animals can be assigned to the 3 diets.

Type of Permutations

- **Example 37:** A ship's captain sends signals by arranging 4 orange and 3 blue flags on a vertical pole. How many different signals could the ship's captain possibly send?

Solution. If the flags were numbered 1, 2, 3, 4, 5, 6 and 7, then the orange flags and blue flags would be distinguishable among themselves. In that case, the ship's captain could send any of **7! = 5,040** possible signals.

The flags are not numbered, however. That is, the four orange flags are not distinguishable among themselves, and the 3 blue flags are not distinguishable among themselves. We need to count the number of distinguishable permutations when the two colors are the only features that make the flags distinguishable. The ship's captain has 4 orange flags and 3 blue flags. Using the formula for the number of distinguishable permutations, the ship's captain could send any of:

$$\frac{7!}{4!\,3!} = 35 \; Possible S$$

Type of Permutations

- **Example 38:** Suppose we toss a gold dollar coin 8 times. What is the probability that the sequence of 8 tosses yields 3 heads (H) and 5 tails (T)?

Solution:

The Product Rule: $\mathbf{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256}$

We can think of choosing (note that choice of word!) r = 3 positions for the heads (H) out of the n = 8 possible tosses. That would, of course, leave then n − r = 8 − 3 = 5 positions for the tails (T). Using the formula for a combination of n objects taken r at a time, there are therefore:

$$\frac{\mathbf{8!}}{\mathbf{3!\,5!}} = \mathbf{56}$$

Distinguishable permutations of 3 heads (H) and 5 tails (T). The probability of tossing 3 heads (H) and 5 tails (T) is thus 56/256 = **0.22.**

Type of Permutations

➢ **Example 39:** In how many ways can 4 married couples attending a concert be seated in a row of 8 seats:

✓ a) if there are no restrictions as to where the 8 people can sit?

✓ b) if each married couple is seated together?

✓ c) if members of the same sex are all seated next to each other

Solution:

a. The Product Rule: **8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 40,320 possible** seating arrangements.

b. Four married couples can be seated in a row of 8 seats in: **4! × 2! × 2! × 2! × 2! = 384 ways** if each married couple is seated together.

c. Four married couples can be seated in a row of 8 seats in**: 4! × 4! × 2 = 1,152 ways** if the members of the same sex are seated next to each other.

Type of Permutations

❖ Type IV: Circular Permutation:

- If clockwise and anti-clock-wise orders are different, and then total number of circular-permutations is given by

$$P(n,r) = (n-1)!$$

- If clock-wise and anti-clock-wise orders same, then total number of circular-permutations is given by

$$P(n,r) = \frac{1}{2}(n-1)!.$$

- **Example 40:** In how many ways can 6 people be seated at a round table?

  Solution: The number of ways will be **(6 – 1)!, or 120.**

Type of Permutations

- **Example 41:** Find the number of ways in which 5 people A,B,C,D,E can be seated at a round table, such that

  (i) A and B must always sit together. And (ii) C and D must not sit together.

  Solution:

  (i) If we wish to seat A and B together in all arrangements, we can consider these two as one unit, along with 3 others. So effectively we've to arrange 4 people in a circle, the number of ways being **(4 – 1)! or 6**.

  But in each of these arrangements, A and B can themselves interchange places in 2 ways.

  Therefore, the total number of ways will be **6 x 2 = 12**.

  (ii) The number of ways in this case would be obtained by removing all those cases (from the total possible) in which C & D are together. The total number of ways will be (5 – 1)! or 24. Similar to (i) above, the number of cases in which C & D are seated together, will be 12. Therefore the required number of ways will be **24 – 12 = 12.**

Type of Permutations

- **Example 42:** In how many ways can 3 men and 3 ladies be seated at around table such that no two men are seated together?

Solution: Since we don't want the men to be seated together, the only way to do this is to make the men and women sit alternately.

We'll first seat the 3 women, on alternate seats, which can be done in (3 – 1)! or 2 ways, (We're ignoring the other 3 seats for now)

That is, if each of the women is shifted by a seat in any direction, the seating arrangement remains exactly the same. i.e why we have only 2 arrangements, Now that we've done this, the 3 men can be seated in the remaining seats in 3! or 6 ways. Note that we haven't used the formula for circular arrangements now. This is so because, after the women are seated, shifting the each of the men by 2 seats, will give a different arrangement. After fixing the position of the women (same as 'numbering' the seats), the arrangement on the remaining seats is equivalent to a linear arrangement.

Therefore the total number of ways in this case will be **2! X 3! = 12.**

Examples on Permutations

- **Example 43:** Suppose that there are eight runners in a race. The winner receives a gold medal, the second place finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?

  Solution: The number of different ways to award the medals is the number of 3-permutations of a set with eight elements.

  Hence, there are **$P(8, 3) = 8 \cdot 7 \cdot 6 = 336$ possible ways** to award the medals.

Examples on Permutations

- **Example 44:** Suppose that a saleswoman has to visit eight different cities. She must begin her trip in a specified city, but she can visit the other seven cities in any order she wishes. How many possible orders can the saleswoman use when visiting these cities?

  Solution: The number of possible paths between the cities is the number of permutations of seven elements, because the first city is determined, but the remaining seven can be ordered arbitrarily.

  Consequently, there are **$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$ ways for the saleswoman** to choose her tour. If, for instance, the saleswoman wishes to find the path between the cities with minimum distance, and she computes the total distance for each possible path, she must consider a total of 5040 paths!

Examples on Permutations

- **Example 45:** How many permutations of the letters ABCDEFGH contain the string ABC ?

  Solution: Because the letters ABC must occur as a block, we can find the answer by finding the number of permutations of six objects, namely, the block ABC and the individual letters D, E, F, G, and H.

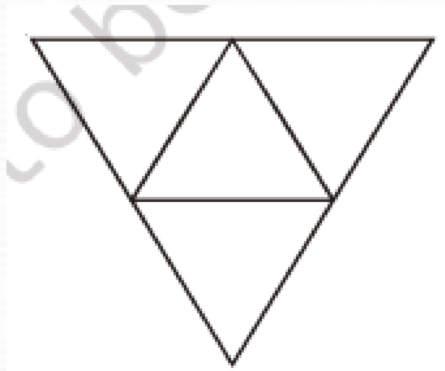
  Because these six objects can occur in any order, there are **6! = 720 permutations** of the letters ABCDEFGH in which ABC occurs as a block.

Examples on Permutations

- **Example 46:** In how many ways can this diagram be coloured subject to the following two conditions?

  (i) Each of the smaller triangle is to be painted with one of three colours: red, blue or green.

  (ii) No two adjacent regions have the same colour.



Solution: These conditions are satisfied exactly when we do as follows: First paint the central triangle in any one of the three colours. Next paint the remaining 3 triangles, with any one of the remaining two colours.

By the fundamental principle of counting, this can be done in $3 \times 2 \times 2 \times 2 = 24$ ways.

Examples on Permutations

- **Example 47**: How many numbers are there between 99 and 1000 having 7 in the units place?

  Solution:

  - First note that all these numbers have three digits. 7 is in the unit's place. The middle digit can be any one of the 10 digits from 0 to 9.
  - The digit in hundred's place can be any one of the 9 digits from 1 to 9.
  - Therefore, by the fundamental principle of counting, there are **10 × 9 = 90** numbers between 99 and 1000 having 7 in the unit's place.

Hundred Place	Tenth Place	Unit Place

Examples on Permutations

- **Example 48**: How many numbers are there between 99 and 1000 having atleast one of their digits 7?

  Solution:

  Total number of 3 digit numbers having atleast one of their digits as 7 = (Total numbers of three digit numbers) – (Total number of 3 digit numbers in which 7 does not appear at all).

  $= (9 \times 10 \times 10) - (8 \times 9 \times 9) = 900 - 648 = 252.$

Examples on Permutations

- **Example 49**: How many ways are there to rearrange the letters in the word COMPUTER?

  Solution: There are 8 distinct letters in the word COMPUTER. Therefore, **8!**

- **Example 50**: How many ways if you are required to keep the vowels in alphabetical order in the word COMPUTER? (In other words, the consonants can be in any order at all, but the vowels must appear in the order EOU, possibly with consonants in between the vowels.)

  Solution: There are 5 consonants and 8 positions to choose from. The number of ways to permutate this is $8 \times 7 \times 6 \times 5 \times 4 = 6720$. There is only 1 way to arrange the vowels in the remaining 3 positions (namely, in the order E, O and U). Therefore, the number of arrangements given the constraints is 6720.

Examples on Permutations

- **Example 51**: How many ways if the consonants must be kept in alphabetical order in the word COMPUTER?

  Solution: 5! ways to permutate the consonants in the word. Therefore, the number of ways to arrange COMPUTER, keeping the consonants in in alphabetical order is **8!/5! = 336.**

- **Example 52**: Find the number of ways in which 10 beads can be arranged to form a necklace.

  Solution: Let us fix the position of one bead. Now, we are left with the arrangement of the remaining, 10 − 1 = 9 beads. These nine beads can arrange themselves in 9P9 = 9! ways. As there is no dependency on the position of beads in a clockwise or anticlockwise manner.

  The required number of ways = 1/2 (9!) = 181440.

Examples on Permutations

- **Example 53**: In how many ways can 5 children be arranged in a line such that (i) two particular children of them are always together (ii) two particular children of them are never together.

  Solution: (i) We consider the arrangements by taking 2 particular children together as one and hence the remaining 4 can be arranged in $4! = 24$ ways. Again two particular children taken together can be arranged in two ways. Therefore, there are **$24 \times 2 = 48$ total ways of arrangement**.

  (ii) Among the $5! = 120$ permutations of 5 children, there are 48 in which two children are together. In the remaining **$120 - 48 = 72$ permutations**, two particular children are never together.

Examples on Permutations

- **Example 54**: If all permutations of the letters of the word AGAIN are arranged in the order as in a dictionary. What is the 49th word?

  Solution: Starting with **letter A**, and arranging the other four letters, there are 4! = 24 words. These are the first 24 words.

  Then **starting with G**, and arranging A, A, I and N in different ways, there are 4!/2!1!1! = 12 words.

  Next the 37th word **starts with I**. There are again 12 words starting with I. This accounts up to the 48th word.

  The 49th word is NAAGI.

Examples on Permutations

- **Example 55**: One hundred tickets, numbered 1, 2, 3, . . . , 100, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti).How many ways are there to award the prizes if

  a) there are no restrictions?

  b) the person holding ticket 47 wins the grand prize?

  c) the person holding ticket 47 wins one of the prizes?

  d) the person holding ticket 47 does not win a prize?

  e) the people holding tickets 19 and 47 both win prizes?

  f) the people holding tickets 19, 47, and 73 all win prizes?

  g) the people holding tickets 19, 47, 73, and 97 all win prizes?

  h) none of the people holding tickets 19, 47, 73, and 97 wins a prize?

  i) the grand prize winner is a person holding ticket 19, 47, 73, or 97?

  j) the people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?

Difference between

Permutation and Combination

Combinations

- In mathematics, a combination is a selection of items from a collection, such that (unlike permutations) **the order of selection does not matter.**
- For example, given three fruits, say an apple, an orange and a pear, there are three combinations of two that can be drawn from this set: an apple and a pear; an apple and an orange; or a pear and an orange.
- More formally, a **k-combination** of a set S is a subset of k distinct elements of S. If the set has n elements, the number of k-combinations is equal to the binomial coefficient:

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1},$$

- Combinations refer to the combination of n things taken k at a time without repetition. To refer to combinations in which repetition is allowed, the terms k-selection, k-multiset, or k-combination with repetition are often used.

Combinations

- **Type 1:** The number of unordered subsets, called a combination of **n** objects taken **r** at a time, is:

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

- We say "***n* choose *r*.**" and denoted by $_nC_r$ or **C(n,r)** or occasionally C_r^n.

Combinations

❖ A. **Type 1:** The number of unordered subsets, called a combination of **n** objects taken **r** at a time, is:

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

- **Proof:** The P (n, r) r-permutations of the set can be obtained by forming the C(n, r) r-combinations of the set, and then ordering the elements in each r-combination, which can be done in P(r, r) ways. Consequently, by the product rule, **P (n, r) = C(n, r)·P(r, r)**
- This implies that

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{n!/(n-r)!}{r!/(r-r)!} = \frac{n!}{r!\,(n-r)!}.$$

- When computing C(n, r), first note that when we cancel out **(n − r)!** from the numerator and denominator of the expression for C(n, r) in Theorem I, we obtain

$$C(n,r) = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\cdots(n-r+1)}{r!}.$$

Combinations

- **Example 56:** How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

  Solution: Because the order in which the five cards are dealt from a deck of 52 cards does not matter, there are:

$$\mathbf{C(52,\ 5)} = \frac{52!}{5!47!} = 2{,}598{,}960$$

  different poker hands of five cards that can be dealt from a standard deck of 52 cards.

  Also,

$$\mathbf{C(52,\ 47)} = \frac{52!}{47!5!} = 2{,}598{,}960$$

  different ways to select 47 cards from a standard deck of 52 cards.

Combinations

- In Example 56 we observed that C(52, 5) = C(52, 47). This is a special case of the useful identity for the number of *r*-combinations of a set given in below property.

- **Property:** Let n and r be nonnegative integers with r ≤ n. Then **C(n, r) = C(n, n − r).**

Proof: We have, $$\mathbf{C(n, r)} = \frac{n!}{r!(n-r)!}$$

Then,

$$C(n, n-r) = \frac{n!}{(n-r)!\,[n-(n-r)]!} = \frac{n!}{(n-r)!\,r!}.$$

Hence proved C(n, r) = C(n, n − r).

Combinations

- **Example 57:** How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?

  Solution: The answer is given by the number of 5-combinations of a set with 10 elements.

$$C(10, 5) = \frac{10!}{5!5!} = 252$$

- **Example 58:** A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job)?

Solution: The number of ways to select a crew of six from the pool of 30 people is the number of 6-combinations of a set with 30 elements, because the order in which these people are chosen does not matter. By Theorem 2, the number of such combinations is

$$C(30, 6) = \frac{30!}{6!24!} = 593{,}775.$$

Combinations

- **Example 59:** A standard deck of cards containing 13 face values (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King) and 4 different suits (Clubs, Diamonds, Hearts, and Spades) to play five-card poker. If you are dealt five cards, what is the probability of getting a "full-house" hand containing three kings and two aces (KKKAA)?

Solution:

Probability of (KKKAA) = $\frac{n(KKKA\ \ )}{(No of Sam\ \ \ \ )}$

No of Sample = C(52, 5) $= \frac{52!}{5!47!}$

No of Sample = C(52, 5) $= \frac{52*51*50*49*47}{5*4*3*2*1} = 2{,}598{,}960$

n(KKKAA) = C(4, 3) . C(4,2) $= \frac{4!}{3!1!} \cdot \frac{4!}{2!2!} = 24$

Probability of (KKKAA) = $\frac{n(KKKA\ \ )}{(No of Sam\ \ \ \ )} = \frac{24}{2{,}598{,}960}$

Combinations

- **Example 60:** In a lottery, each ticket has 5 one-digit numbers 0-9 on it.

  a) You win if your ticket has the digits in any order. What are your chances of winning?

  b) You would win only if your ticket has the digits in the required order. What are your chances of winning?

  Solution:

  There are 10 digits to be taken 5 at a time.

  a) Using the formula: $C(10, 5) = \frac{10!}{5!5!} = 252$

  The chances of winning are 1 out of 252.

  b) Since the order matters, we should use permutation instead of combination.

  $P(10, 5) = 10 \times 9 \times 8 \times 7 \times 6 = 30240$

  The chances of winning are 1 out of 30240.

Combinations

❖ **Type II: Combination with Repetition:** The number of ways to fill **r** slots from **n** categories with repetition allowed is:

$$C(r + n - 1, r) = C(r + n - 1, n - 1).$$

➢ **Example 61:** How many ways can I fill a box holding 100 pieces of candy from 30 different types of candy?

Solution: Here r = 100, n = 30, Therefore n-1 = 30-1 = 29

$$C(r + n - 1, r) = C(r + n - 1, n - 1)$$

$$C(100 + 29, 100) = 129!/(100!29!)$$

Different ways to fill the box.

Combinations

- **Example 62:** What is the number of ways of choosing 4 cards from a pack of 52 playing card? In how many of these

  a) Four cards are of the same suit

  b) Four cards belong to four different suit

  c) Are face cards

  d) Two are red cards and two are black cards

  e) Cards of same colour

  Solution:

Combinations

➢ **Solution of Example 62:** What is the number of ways of choosing 4 cards from a pack of 52 playing card? In how many of these

a) Four cards are of the same suit

Solution:

Cards	**Total**	**No. of cards to be chosen**	**No. of ways**
Diamond	13	4	C(13, 4)
Spade	13	4	C(13, 4)
Heart	13	4	C(13, 4)
Club	13	4	C(13, 4)

Since, there are different cases, So, we add number of ways

The required number of ways choosing four cards of the same suit

$= C(13, 4) + C(13, 4) + C(13, 4) + C(13, 4).$

$= 4 * C(13, 4) \quad = 4 * \frac{13!}{4!9!} = 2860$ ways

Combinations

- **Solution of Example 62:** What is the number of ways of choosing 4 cards from a pack of 52 playing card? In how many of these

  b) Four cards belong to four different suit

Solution:

Cards	Total	No. of cards to be chosen	No. of ways
Diamond	13	1	C(13, 1)
Spade	13	1	C(13, 1)
Heart	13	1	C(13, 1)
Club	13	1	C(13, 1)

Since, there are some cases, So, we multiply number of ways

= C(13, 1) * C(13, 1) * C(13, 1) *C(13, 1).

= $(C(13, 1))^4$ = $(13)^4$ = 13 * 13 * 13 * 13 = 28561 ways.

Combinations

- **Solution of Example 62:** What is the number of ways of choosing 4 cards from a pack of 52 playing card? In how many of these

  c) Are face cards

  Solution:

  Number of face cards in one suit = 3

  Total number of face cards = 4 * 3 = 12 i.e n =12

  Number of card to be selected are = 4 i.e r = 4

  Number of face cards chosen are = C(12,4) = 495 ways.

Combinations

- **Solution of Example 62:** What is the number of ways of choosing 4 cards from a pack of 52 playing card? In how many of these

  d) Two are red cards and two are black cards

Solution:

Cards	Total	No. of cards to be chosen	No. of ways
Red	26	2	C(26, 2)
Black	26	2	C(26, 2)

Total number of ways of choosing two are red cards and two are black cards $= C(26,2) * C(26,2) = (C(26,2))^2 = 105625$.

Combinations

- **Solution of Example 62:** What is the number of ways of choosing 4 cards from a pack of 52 playing card? In how many of these

e) Cards of same colour

Solution:

Cards	Total	No. of cards to be chosen	No. of ways
Red	26	4	C(26, 4)
Black	26	4	C(26, 4)

Since Choosing Red or Black, they are different cases

So, we add numbers of ways

Total number of ways of selecting four of same color

$= C(26,4) + C(26,4) = 2(C(26,4)) = 29900.$