Set Theory

A set is a collection of objects.
(The objects are also called elements or members of the set.)
If x is an element of the set S, we write: $x \in S$
If x is **not** an element of the set S, we write: $x \notin S$

Examples:
1. The set V of vowels in the English alphabet:
V = {a, e, i, o, u}.

2. Let P be the set of positive integers less than 100:
P = {1, 2, 3,…, 99}.

3. We can also use predicate notation for P:
P = {*x* | *x* is a positive integer less than 100}

Set Theory

Equal Sets: Two sets are equal if and only if they contain the same elements. Example: The sets {1, 3, 5} and {3, 5, 1} are equal. The set {1, 1, 3, 3, 3, 5} is also equal to {1, 3, 5}.

A set A is said to be a subset of B, and we note ,
if and only if every element of A is also an element of B.

$$A \subseteq B \Leftrightarrow \forall x\,(x \in A \rightarrow x \in B)$$

Every set is a subset of itself.

The null (or empty) set ø is a subset of all sets.

A is said to be a proper subset of B, and we denote , $A \subset B$
if and only if $A \subseteq B \ and\ A \neq B$

Set Theory

Cardinality: Let ‘S’ be a set. If there are exactly ‘n’ distinct elements in S, we say that S is a finite set, and that ‘n’ is the cardinality of S. We write $|S| = n$.
Example: Let A be the set of letters in the English alphabet:
$|A| = 26$.
A set is said to be **infinite** if it is not finite.
Example: The set of positive integers is infinite.

Power Set: The power set of a set S, denoted by P(S), is the set of all subsets of S. If the set S has ‘n’ elements, its power set will have $\mathbf{2^n}$ elements.

Cartesian Product: Let A and B be two sets. The Cartesian product of A and B, denoted by $(A \times B)$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.
$A \times B = \{(a, b) \mid a \in A \land b \in B\}$.

Set Theory

Example: Let A = {1, 2}, and B={a, b, c};
A×B = {(1, a), (1, b), (1, c), (2, a), (2, b), (2,c)}
B×A = {(a, 1), (b, 1), (c, 1), (a, 2), (b, 2), (c,2)}

The Cartesian product of the sets A1, A2, …, An, denoted by A1×A2×… ×An, is the set of ordered n-tuples (a1, a2, …, an), where ai belongs to Ai for i=1, 2, …, n**.** A1 × A2 × …× An = {(a1, a2, …, an) | ai ∈Ai for i=1, 2, …, n}.

If A = {0, 1}, B = {1, 2}, C = {0,1}; then what is A×B×C ?

A×B×C = {(0,1,0), (0,1,1), (0,2,0), (0,2,1), (1,1,0), (1,1,1), (1,2,0), (1,2, 1)}.

Set Theory

Operations on Sets

Union

The union of A and B, written $A \cup B$, is the set whose elements are the elements of A or B or both.

In the predicate notation the definition is

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Examples: Let $K = \{a, b\}$, $L = \{c, d\}$ and $M = \{b, d\}$, then

$K \cup L = \{a, b, c, d\}$

$K \cup M = \{a, b, d\}$

$L \cup M = \{b, c, d\}$

Intersection

The intersection of A and B, written $A \cap B$, is the set whose elements are just the elements of both A and B. In the predicate notation the definition is

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Set Theory

Examples: Let $K = \{a, b\}$, $L = \{c, d\}$ and $M = \{b, d\}$, then

$K \cap L = \varphi$ $K \cap M = \{b\}$ $L \cap M = \{d\}$.

Difference

The difference of A and B, written A – B, is the set whose elements are the elements of A which are not in B.

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$.

Examples: Let $K = \{a, b\}$, $L = \{c, d\}$ and $M = \{b, d\}$, then

$K - L = \{a, b\}$

$K - M = \{a\}$

$L - M = \{c\}$

Symmetric difference

The symmetric difference of sets A and B, denoted by $A \oplus B$, consists of those elements which belong to 'A or B' but not 'A and B'.

$(A \oplus B) = (A \cup B) - (A \cap B)$. We can also write

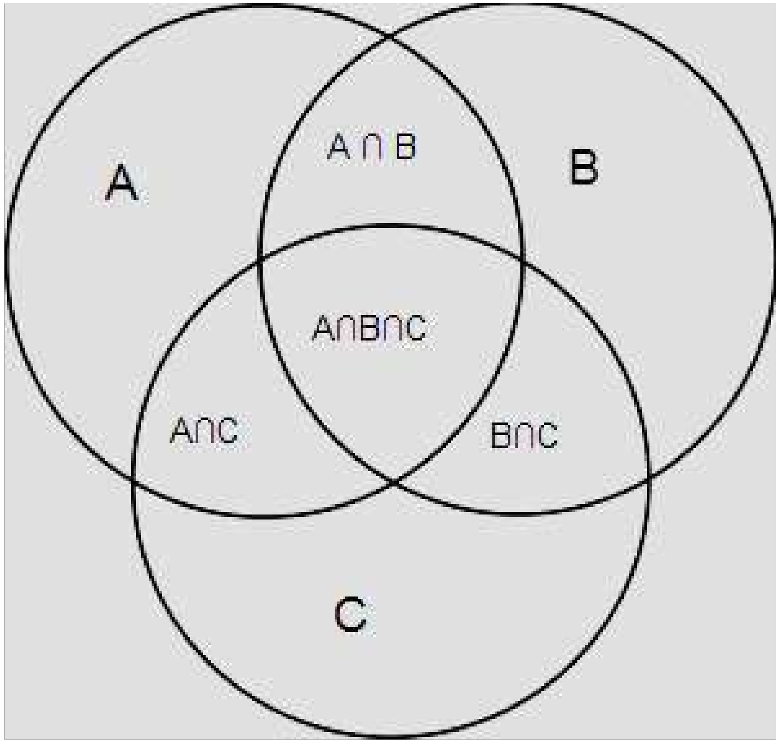
$(A \oplus B) = (A - B) \cup (B - A)$.

Set Theory

Complement: The complement of a set A, written A^c, which is the set consisting of everything not in A. $A^c = \{x \mid x \notin A\}$. The same can also be written as $A^c = U - A$, where U is the universal set.

Venn Diagram

A Venn diagram is a diagram constructed with a collection of circles drawn on a plane. Each Venn diagram begins with a rectangle representing the universal set. Then circles represent the sets. Any values that belong to more than one set will be placed in the sections where the circles overlap.



Set Theory

Mutually Exclusive and Exhaustive Sets

We say that a group of sets is **exhaustive** of another set if their union is equal to that set. For example, if $A \cup B = C$ we say that A and B are exhaustive with respect to C.

We say that two sets A and B are **mutually exclusive** if, $A \cap B = \emptyset$ that is, the sets have no elements in common.

The Principle of Inclusion and Exclusion

The Principle of Inclusion and Exclusion allows us to find the cardinality of a union of sets by knowing the cardinalities of the individual sets and all possible intersections of them. The basic version of the Principle of Inclusion and Exclusion is that for two finite sets A and B, it holds that $|A \cup B|=|A|+|B|-|A \cap B|$.

If there are three finite sets:

$|A \cup B \cup C|=|A|+|B|+|C|-|A \cap B|-|A \cap C|-|B \cap C|+|A \cap B \cap C|$.

Set Theory

Example: Let $A=\{a, b, c, d\}$ and $B=\{c, d, e, f\}$ be two sets with cardinalities 4.

$A \cup B=\{a, b, c, d, e, f\}$, $A \cap B = \{c, d\}$.

$|A \cup B|=|A|+|B|-|A \cap B| = 4+4-2 = 6$.

Q1) Suppose that in a certain exam 200 students appear for Mathematics, 50 appear for Physics, and 20 appear for both Mathematics and Physics. How many have appeared for Mathematics or Physics?

Answer: Let 'M' be the set of students appear for Mathematics and 'P' be the set of students appear for Physics. It is given that $|M|=200$ and $|P|=50$ and $|M \cap P|=20$. We need to find $|M \cup P|$. According to the principle of inclusion exclusion, $|M \cup P|=|M|+|P|-|M \cap P|=200+50-20=230$.

Set Theory

Q2) A certain computer center employs 100 programmers. Of these 47 can program in FORTRAN, 35 in PASCAL, 20 in COBOL, 23 in FORTRAN and PASCAL, 12 in COBOL and FORTRAN, 11 in PASCAL and COBOL and 5 in all three. How many employees can not program in any of these 3 languages.

Answer: First we need to find the number of people who can program in at least one language. That is $|F \cup C \cup P|$.

$|F \cup C \cup P|=|F|+|C|+|P|+|F \cap C \cap P|-(|F\cap P|+|F\cap C|+|P\cap C|)$

$= 47 + 20 + 35 + 5-(23 + 12 + 11) = 61$.

Number of people who can program in **none** of the languages = 100-61=39.

Q3) A survey shows that 58% of Indians like coffee and 75% like tea. What is the percentage of Indians liking both coffee and tea?

Set Theory

Answer: consider that 100 people are there. 58 of them like coffee and 75% of them like tea.

$|C \cup T| = |C| + |T| - |C \cap T|$

$100 = 58 + 75 - |C \cap T|$

$|C \cap T| = 33$. So the percentage of people liking coffee and tea is $(33/100)*100 = 33\%$.

Assignment:

Q4) In a class of 100 students, 39 play Tennis, 58 play cricket, 32 play hockey, 10 play cricket and hockey, 11 play hockey and tennis and 13 play tennis and cricket. What is the number of students who play only cricket?

Set Theory

Answer: Number of students who play only cricket is
$\mathbf{|C| - (|C \cap H| + |C \cap T|) + |C \cap H \cap T|}$.
First we need to find $|C \cap H \cap T|$.
$100 = 39 + 58 + 32 - (13 + 10 + 11) + |C \cap H \cap T|$.
$|C \cap H \cap T| = 5$.
Number of students who play only cricket
$= 58 - (10 + 13) + 5 = 40$.

Q5) In a senior class of 100 students 12 failed math, 10 failed chemistry, and 2 failed both. How many passed both?

Multiset Operation

For example, writing finite multisets (those with finite domains and finite multiplicities for all elements) in their usual notations, if

$f=\{a,a,b,b,b,c,d,d\}$ and $g=\{b,b,c,c,c,d,d,e\}$, then

$f\cup g=\{a,a,b,b,b,c,c,c,d,d,e\}$

$f\cap g=\{b,b,c,d,d\}$

$f+g=\{a,a,b,b,b,b,b,c,c,c,c,d,d,d,d,e\}$

$f-g=\{a,a,b\}$

MAthematical induction

Example 2: **Show that $1 + 3 + 5 + \ldots + (2n-1) = n^2$**

Solution:

Step 1: Result is true for $n = 1$

That is $1 = (1)^2$ (True)

Step 2: Assume that result is true for $n = k$

$1 + 3 + 5 + \ldots + (2k-1) = k^2$

Step 3: Check for $n = k + 1$

i.e. $1 + 3 + 5 + \ldots + (2k-1)+(2(k+1)-1) = (k+1)^2$

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We can write the above equation as,

$1 + 3 + 5 + \ldots + (2k-1) + (2(k+1)-1) = (k+1)^2$

Using step 2 result, we get

$k^2 + (2(k+1)-1) = (k+1)^2$

$k^2 + 2k + 2 - 1 = (k+1)^2$

$k^2 + 2k + 1 = (k+1)^2$

$(k+1)^2 = (k+1)^2$

L.H.S. and R.H.S. are same.

So the result is true for $n = k+1$

By mathematical induction, the statement is true.