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* Relation

- **Cartesian Product**

Let A and B be 2 non-empty sets. The cartesian product is denoted by $A \times B$ and defined as.

$$A \times B = \{(a,b) / a \in A \text{ and } b \in B\}$$

eg Let $A = \{1,2,3\}$ $B = \{x,y\}$

$\therefore A \times B = \{(1,x), (1,y), (2,x), (2,y), (3,x), (3,y)\}$

Here $A \times B \neq B \times A$ is not commutative.

$[\because (1,x) \neq (x,1)]$

- Theorem 1

$|A| = m$, $|B| = n$ then $|A \times B| = m \times n$

- Theorem

If A, B, C are sets then

i) $A \times (B \cap C) = (A \cap B) \times (A \times C)$

ii) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

iii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

iv) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

- **Relation**

A relation from A to B is any subset of $A \times B$. It is denoted by $R: A \rightarrow B$

eg $A = \{x,y,z\}$ $B = \{1,2\}$

$A \times B = \{(x,1), (x,2), (y,1), (y,2), (z,1), (z,2)\}$

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Here $R_1 : A \rightarrow B$, $R_2 : A \rightarrow B$

$R_1 = \{(x,1), (y,1), (z,1)\}$

$R_2 = \{(x,1), (x,2)\}$

Here R_1 & R_2 are subsets of $A \times B$

$R_3 : B \rightarrow A$

eg. $R_3 = \{(2,x), (2,y), (2,z)\}$

Here R_3 is subset of $B \times A$

- Important

i) If $(a,b) \in R$ it is denoted by aRb

ii) $R : A \rightarrow B$ then $R \subseteq A \times B$

iii) If R is relation from A to B, then "a" in ordered pairs $(a,b) \in R$ is called domain of R.

$D(R) = \{a \mid (a,b) \in R\}$

also "b" in ordered pairs $(a,b) \in R$ is called range of R

$R(R) = \{b \mid (a,b) \in R\}$

iv) The null set is subset of $A \times B$

$\therefore$ ϕ is relation called null/empty relation

* Matrix representation of Relation

$A = \{a_1, a_2, a_3, \ldots a_n\}$

$B = \{b_1, b_2, b_3, \ldots b_m\}$

relation matrix of R is denoted by $M_R = [m_{ij}]_{n \times m}$ and defined by

$$m_{ij} = \begin{cases} 0, & \text{if } (a_i, b_j) \notin R \\ 1, & \text{if } (a_i, b_j) \in R \end{cases}$$

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eg. $A = \{1, 2, 3, 4\}$ ~~st~~. $R: A \to A = \{(a, b) / a < b\}$

$R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

$$\therefore M_R = \begin{array}{cc} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

* **Diagraph**

A is not empty set

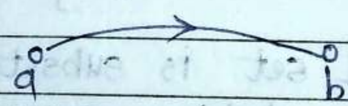
R is relation on A

- Rules

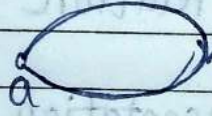
i) The elements of set A are represented by small circles or point (∘) these elements are called as vertices or nodes.

ii) If $(a, b) \in R$ then vertices a and b are joined by a continuous arc with an arrow from a to b. These arcs are known as edges of graph

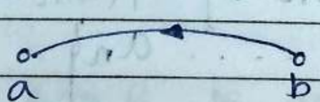
i.e. i) if aRb then



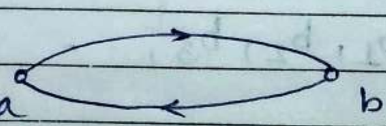
ii) if aRa then



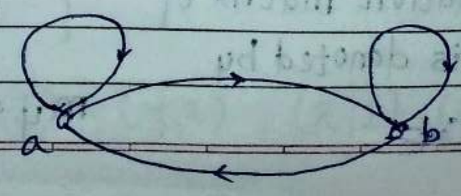
iii) bRa



iv) $aRb \wedge bRa$



v) $aRb \wedge bRa \wedge aRa \wedge bRb$



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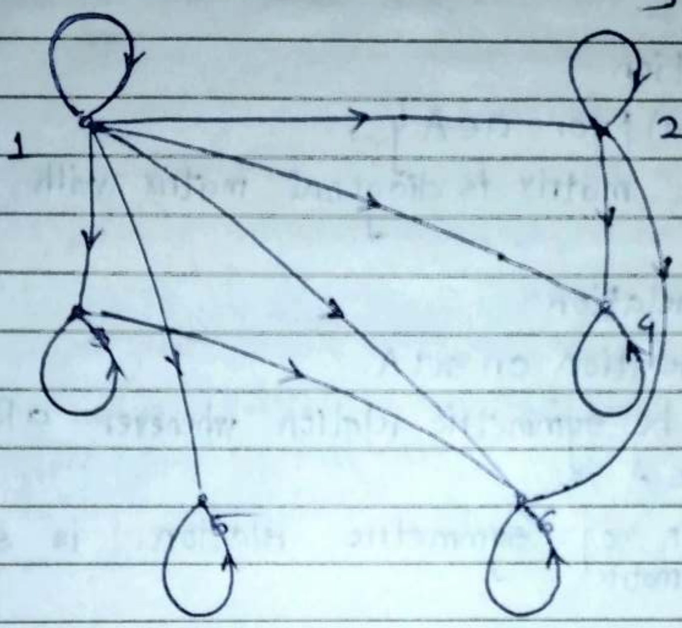
DATE (a divides b)

Q. $A = \{1, 2, 3, 4, 5, 6\}$ and aRb iff ~~a/b~~ b/a. Draw diagraph of R.

→ $R = \{(1,1), (2,2), (4,4), (3,3), (5,5), (6,6), (1,2), (1,3), (1,4), (1,5), (1,6), (2,4), (2,6), (3,6)\}$



Q. R is a relation of real number defined as $x \leq y$

$R = \{(x,y) \mid x, y \in R \text{ and } x \leq y\}$

Check whether it is equivalence relation.

→ Here following relation is reflexive as it consists of ordered pairs (a,a)

It is not symmetric becoz it doesn't consist of (a,b) and (b,a)

It is ~~not~~ transitive because since $(x,y) \in R$ ~~at~~ for $x \leq y$

also $(y,z) \in R$ for $y \leq z$

$\therefore (x,z) \in R$ and also $x \leq z$

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* **Reflexive relation**

A relation R is said to be reflexive relation if $(a,a) \in R,\ \forall a \in A$

i) The relation matrix is identity matrix.

* **Identity relation**

$I_A = \{(a,a) \mid \text{for } a \in A\}$

i) The relation matrix is diagonal matrix with 1 and 0

* **Symmetric relation**

Let R be relation on set A.

R is said to be symmetric relation whenever aRb then bRa for $a,b \in A$.

i) The relation matrix of symmetric relation is symmetric matrix

* **Compatible relation**

A relation on R is said to be compatible if it is reflexive and symmetric

i) The relation matrix of compatible relation is symmetric matrix with diagonal elements 1

* **Asymmetric relation**

A relation ~~on~~ R on set A is said to be assymmetric relation whenever aRb then $b \not R a$

[If atleast one element Symmetric, then it is not Asymmetric]

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* Antisymmetric relation

- Here A relation R on set A is said to antisymmetric if aRb and bRa then a=b.

- if aRb and bRa but $a \neq b$ then it is not antisymmetric.

* eg. Let $A = \{1, 2, 3\}$ then

$R_1 = \{(1,1), (2,2)\}$ is symmetric, antisymmetric

$R_2 = \{(1,2), (2,1)\}$ is symmetric but not antisymmetric relation.

$R_3 = \{(1,1), (1,2), (2,2)\}$ is antisymmetric but not asymmetric.

The relation matrix of antisymmetric relation is never symmetric matrix.

* Transitive Relation

R be a relation on set A.

R is transitive if aRb and bRc then aRc

eg. $A = \{x, y, z\}$

$R = \{(x,y), (y,z), (x,z)\}$

also R is transitive if only aRb and there no bRc

eg. $R = \{(x,y), (z,y)\}$

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* Equivalence relation

Let R be a relation on set A.

R is equivalence iff

R is reflexive, symmetric, and transitive

SCS

Weakness :

Unrealistic — I tend to ~~have~~ ~~[illegible]~~ put pressure on

to do everthing ~~that~~ right what I encounter. But

it just isn't realistic for me to solve every pr

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behavior with them gets shaped by it

* Properties of Equivalence relation.

1) If R_1 and R_2 are equivalence relations on set A then

→ $R_1 \cap R_2$ is equivalence

→ $R_1 \cup R$ is not necessarily equivalence

* Equivalence classes of x is denoted by $[x]$

$$[x] = \{y \mid y \in A \text{ and } (x,y) \in R\}$$

Let R be an equivalence relation on set A. The equivalence class of $a \in A$ is denoted by $[a]_R$ and defined as

$$[a]_R = \{x \in A \mid xRa\}$$

= The set of those elements of A which are related to a.

* Closure of a Relation

i) Reflexive closure

Let R be a relation on set which is not reflexive.
A relation $R_1 = R \cup \Delta$ is called reflexive closure of R if $R \cup \Delta$ is the smallest reflexive relation containing R.

If $A = \{a, b, c, d\}$ then $\Delta = \{(a,a), (b,b), (c,c), (d,d)\}$

eg. $A = \{1, 2, 3\}$ R_1, R_2, R_3 are relations on set A. Find reflexive closure of them resp. $R_3 = \{(3,1), (1,3), (2,3)\}$

$R_1 = \{(1,1), (2,1)\}$ $R_2 = \{(1,1), (2,2), (3,3)\}$

→ $\Delta = \{(1,1), (2,2), (3,3)\}$

Reflexive closure of R_1 is $R = R_1 \cup \Delta = \{(1,1), (2,1), (2,2), (3,3)\}$

→ " — R_2 is $R = R_2 \cup \Delta = R_2$

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2) Symmetric closure.

R is a relation on set A & it is not symmetric.

A relation $R = R \cup R^{-1}$ is called the symmetric closure of R if $R \cup R^{-1}$ is smallest symmetric relation containing R.

eg. Find symmetric closure of foll^n $A = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,1)\}$

$R_2 = \{(1,2), (2,1), (3,2), (2,2)\}$

$R_3 = \{(1,1), (2,2), (3,3)\}$

→ Here

$R_1 = \{(1,1), (2,1)\}$ $R_1^{-1} = \{(1,1), (1,2)\}$

$R_1 \cup R_1^{-1} = S(R_1) = \{(1,1), (2,1), (1,2)\}$

Similarly

$S(R_2) = \{(1,2), (2,1), (3,2), (2,3), (2,2)\}$

3) Transitive closure.

R is a relation on set A & it is not transitive.
Transitive closure of a relation R is smallest transitive relation containing R.

eg. $A = \{1, 2, 3, 4, 5\}$

$R = \{(1,2), (3,4), (4,5), (4,1), (1,1)$

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→ $R^* = \{(1,2), (2,1), (3,4), (4,1), (4,5), (5,1), (1,1), (3,1)\}$

Q. $A = \{1, 2, 3, 4\}$

$R = \{(1,2), (2,1), (2,3), (3,4)\}$

→

$r(R) = \{(1,2) (2,1) (2,3) (3,4) (1,1) (2,2) (3,3) (4,4)\}$

$s(R) = \{(1,2) (2,1) (2,3) (3,2) (3,4) (4,3)\}$

$R^* = \{(1,2) (2,1), (2,3), (2,4), (3,4), (1,1), (1,3), (1,4), (4,1), (3,1)\}$

* Warshall's Algorithm to find Transitive closure.

Step 1: We have $|A| = n$

∴ We require $W_0, W_1, \ldots W_n$ Warshall sets

W_0 = Relation Matrix of R - M_R

Step 2: To find transitive closure of relation R on set A, with $|A| = n$

W_k from W_{k-1} is computed as follows:

i) Copy 1 of all entries in W_{k-1}, where there is a 1 in W_{k-1}

ii) Find row numbers $p_1, p_2, \ldots$ for which there is 1 in column k in W_{k-1} and the column numbers $q_1, q_2, \ldots$ for which there is 1 in row of W_{k-1}.

iii) Mark entries in W_k as 1 for (p_i, q_i). If there are not already 1.

iv) Stop when W_n is obtained and it is req. transitive closure of R

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Q. To find Transitive closure of R by Warshall's algo. Where

$A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(x, y) \mid (x - y) = 2\}$

$\longrightarrow R = \{(3,1), (4,2), (5,3), (6,4), (1,3), (2,4), (3,5), (4,6)\}$

Here $|A| = 6$

$\therefore$ We want $W_0, W_1, W_2, W_3, W_4, W_5, W_6$

Now $W_0 = M_R$

$$\therefore W_0 = \begin{array}{c|c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Step 2: Find W_1

W_1 from W_0 consider C_1 & R_1 (first column & row)

In C_1 1 is present at R_3

R_1 1 is present at C_3

Add new entry at R_3, C_3

$$\therefore W_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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Now W_2 consider C_2 & R_2

In C_2 1 is at R_4

R_2 1 is at C_4

$\therefore$ New entry at R_4, C_4

$$W_2 = \begin{matrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{matrix}$$

for W_3 consider C_3 & R_3

In C_3 1 is at R_1, R_3, R_5

R_3 1 is at C_1, C_3, C_5

$\therefore$ Add new entry at

(R_1, C_1), (R_1, C_3), (R_1, C_5), (R_3, C_1), (R_3, C_3), (R_3, C_5), (R_5, C_1), (R_5, C_3), (R_5, C_5)

$$W_3 = \begin{matrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{matrix}$$

Similarly ~~for~~ keep solving till we get W_6 which is transitive closure of R.

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* POset (Partially ordered set)

A relation R on A is said to be partialy ordered iff R is reflexive and antisymmetric and transitive relation.

It is denoted by (A,R) or (A $\leq$)

'$\leq$' is a partially ordered relation.

1) Comparable elements:

(A $\leq$) be Poset. 2 elements (a,b) in A are said to be comparable elements if $a \leq b$ or $b \leq a$. 2 elements a and b are non comparable if neither $a \leq b$ nor $b \leq a$

ii) Totaly ordered set:

Let A be any non empty set. The set A is called linearly ordered set if every pair of elements in A are comparable.

i.e. for any $a, b \in A$ either $a \leq b$ or $b \leq a$

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* Partitions of a Set

Definition:

Let A be any non-empty set. A set $P = \{A_1, A_2, \ldots, A_n\}$ of non empty subsets of A is called a partition of set A if

i) $A_1 \cup A_2 \cup \ldots A_n = A = \bigcup_{i=1}^{n} A_i$

Set A is union of set $A_1, \ldots A_n$

ii) $A_i \cap A_j = \Phi$ i.e. All sets A_i are mutually disjoint.

Partition of set A is denoted by π

element of partition — block

Rank of partition — no. of blocks ($r(\pi)$)

eg. If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A_1 = \{1, 4, 9\}$ $A_2 = \{2, 6, 8, 10\}$ $A_3 = \{3, 5, 7\}$

The set $P = \{A_1, A_2, A_3\}$ is such that

i) A_1, A_2, A_3 are non empty sets.

ii) $A = A_1 \cup A_2 \cup A_3$

iii) $A_1 \cap A_2 = \Phi$ $A_1 \cap A_3 = \Phi$ $A_2 \cap A_3 = \Phi$

Hence $\{A_1, A_2, A_3$ form a partition for set A.

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Q S - Se I
R = {(a, b) | a − b is odd +ve integer}.
check for Poset.

→

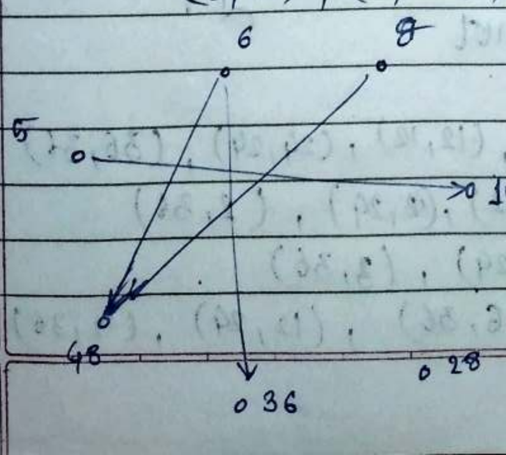
Here for Poset
Reflexive we want ordered pairs of all integers.
Here we want (a, a) ∀ a ∈ I
but a − a = 0 i.e neither +ve nor odd.
∴ R is not reflexive

Symmetric will also be same.

* Hasse diagram
1) Elements of relation R are called vertices and denoted by points
2) All Loop are ommited as relation is reflexive on Poset.
3) If aRb a ≤ b then join a to b by single line called an edge the vertex b appears above level of vertex a. ∴ Arrows may be omitted from the edges in Hasse diagram.
4) Delete all edges that are implied by transitive relation.

Q. A = {5, 6, 8, 10, 28, 36, 48}
R = {(a, b) | a is divisor of b}

→ R = {(5,5), Reflexive,
(5, 10), (6, 36), (6, 48), (8, 48)}



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Partition of set A is denoted by π

element of partition — block
Rank of partition — no. of blocks
($r(\pi)$)

eg. If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

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The set $P = \{A_1, A_2, A_3\}$ is such that

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Hence $\{A_1, A_2, A_3$ form a partition for set A.

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Q S - Se I

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check for Poset.

→

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Here we want $(a,a)\ \forall a \in I$

but $a-a=0$ i.e neither +ve nor odd.

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Symmetric will also be same.

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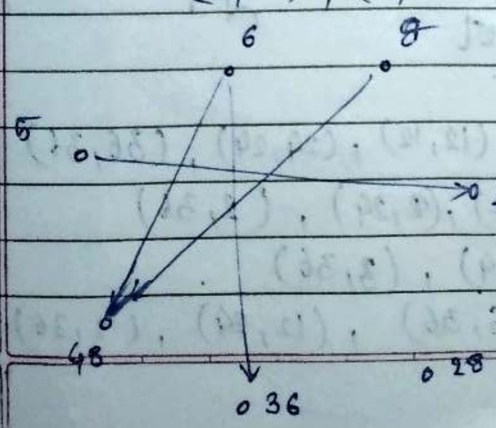
Q. $A = \{5, 6, 8, 10, 28, 36, 48\}$

$R = \{(a,b) \mid a$ is divisor of $b\}$

→ $R = \{(5,5)$, Reflexive,

$(5,10), (6,36), (6,48), (8,48)\}$



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$\wedge$ greatest lowest bound
$\vee$ least ~~lower~~ upper bound.

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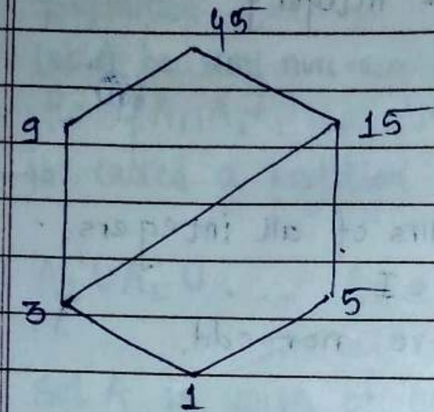
Q Consider PO relation on set S



1, 3, 5, 15, 9, 45

It is a lattice

ordered pairs

a	b	glb	lub
1	3	1	3
1	5	1	5
1	9	1	9
1	15	1	15
1	45	1	45
3	5	1	15
3	9	3	9
3	15	3	15
3	45	3	45
5	9	1	45
5	15	5	15
5	45	5	45
9	15	3	45
9	45	9	45
15	45	15	45

chain

$\{\{1, 3, 9, 45\} \leq\}$

$\{\{1, 3, 5, 45\} \leq\}$

$\{\{1, 5, 15, 45\} \leq\}$

antichain

$\{\{3, 5\} \leq\}$ $\{\{9, 15\} \leq\}$ $\{\{5, 9\} \leq\}$

Q $X = \{2, 3, 6, 12, 24, 36\}$
$x < y$ if x divides y. Draw hasse & find chain & anti chain
Check if it is lattice or not

$R = \{(2,2), (3,3), (6,6), (12,12), (24,24), (36,36)$
~~(2,4)~~, $(2,6), (2,12), (2,24), (2,36)$
$(3,6), (3,12), (3,24), (3,36)$
$(6,12), (6,24), (6,36), (12,24), (12,36)\}$

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* Equivalence class

$A = \{1, 2, 3, 4, 5\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (4,5), (5,4)\}$

Here $[1]_R = \{1,2\}$ $[2]_R = \{1,2\}$

$[3]_R = \{3\}$ $[4]_R = \{4,5\}$ $[5]_R = \{4,5\}$

There can be 1 partition where $R = A \times A$
i.e. min. no. of partions.

∴ Partitions are

$P_1 = \{1,2\}$ $P_2 = \{3\}$ $P_3 = \{4,5\}$

we can find relation from partions by cross product of partition with itself.

* Partially ordering relation.

R is a relation on A such that $R = A \times A$ then R is not Partial ordering relation.

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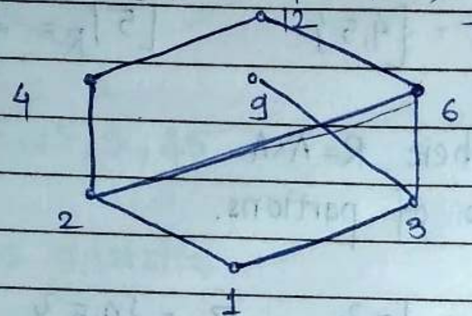
Q. Let $A = \{1, 2, 3, 4, 6, 9, 12\}$
aRb if a divides b
Draw Hasse diagram and check whether it is lattice.

→ $R = \{(1,1), (1,2), (1,3) \ldots$ all ones
$(2,4), (2,6), (2,12), (3,6), (3,9), (3,12)$
$(4,12), (6,12)\}$



here no lub for elements
with 9
∴ not a lattice.

a	b	glb	lub
1	2	1	2
1	3	1	3
1	4	1	4
1	6	1	6
1	9	1	9
1	12	1	12
2	3	1	12
2	4	2	4
2	6	1	12
2	9	1	X
2	12	2	12
3	4	1	12
3	6	3	6
3	9	3	9
3	12	3	12
4	6	1	12
4	9	1	X
4	12	4	12
6	9	3	X
6	12	6	12
9	12	3	X

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Q. Prove
is divisible by 25 by Mathematical induction

→ Here

$P(n) = 7^{2n} + (2^{3n-3})(3^{n-1})$

Step 1: Basis of induction

Put $n=1$,

$\therefore P(1) = 7^2 + 1 = 50$

Here $P(1)$ is divisible by 25

Step 2: Induction hypothesis:

Put $n=k$

$\therefore P(k) = 7^{2k} + (2^{3k-3})(3^{k-1})$

Let $P(k)$ be true

$= 7^{2k} + ((2^3).(3))^{k-1}$

Step 3: Induction step

Put $n=k+1$

$P(k+1) = 7^{2+2k} + (2^{3k})(3^k)$

$= 7^2.7^{2k} + 2^{3k}.3^k$

$= 7^2.7^{2k} + (2^3.3)^k$

$= 7^2.7^{2k} + 24(2^3.3)^{k-1}$

$= (25+24)(7^{2k}) + 24(2^3.3)^{k-1}$

$= 25.7^{2k} + 24(7^{2k} + (2^3.3)^{k-1})$

Now $P(k)$ is divisible by 25

and 25.7^{2k} is also

$\therefore P(k+1)$ is also divisible by 25

$\therefore P(n)$ is divisible by 25

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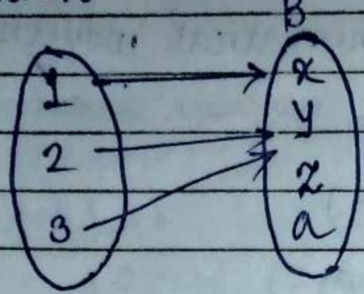
* Function



domain = (1, 2, 3)
co-domain = (x, y, z, a)
range = (x, y)

$f : A \longrightarrow B$ or $f(a) = b$ such that $\forall a \in A$ there is a $b \in B$

~~A is~~ ~~A~~ single relation from A is there with B. But vice versa may not be true

i.e (1, x), (2, y), (3, y)

~~In~~.

- Here every function is a relation but vice versa is not true.

- A function is a relation which describes that there should be only one output for each input.

• Types of function

1) One to One (Injective function)

Here $f : P \longrightarrow Q$ for each element of P there is a distinct element of Q.

ii) Many to one :

Here $f : P \longrightarrow Q$ for some elements in P there is a common element of Q.

iii) Onto function (Surjective)

Here $f : P \longrightarrow Q$ for all elements in Q there is an element from P

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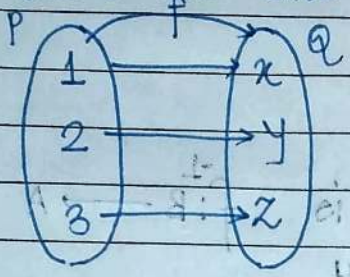
Remember all conditions are considering there is a relation ~~from~~ of A each element from A

iv) Into function

Here $f: P \to Q$ such that there are some elements in Q which ~~are~~ have no element from P.

v) One to one onto (Bijective)



* Composite function.

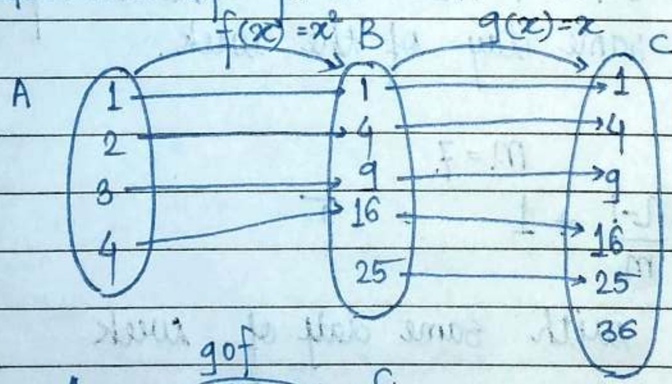
$f: A \to B$ $g: B \to C$

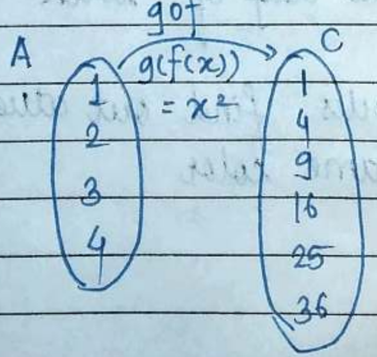
~~gof~~ $\longrightarrow$

$gof : A \to C$

$(gof)(a) = g(f(a)) \neq c \quad \forall a \in A$

- gof is defined only when range of f is subset of domain of g.





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- Identity fxn

$f: A \longrightarrow A$ is an I fxn if $f(x) = x \quad \forall x \in A$

- Constant fxn

$f: A \longrightarrow B$ is a const. fxn
if $f(x) = c \quad \forall x \in A$

- Inverse function

$f: A \longrightarrow B$ then inverse is $f^{-1}: B \longrightarrow A$
such that if $f(x) = y$
$f^{-1}(y) = x$
also $f^{-1}(f(x)) = x$

* Pigeonhole principle
if there are n pigeons & m pigeonholes $(n<m)$
then 2 pigeons will have same hole.

Q. 30 people are selected show that atleast 5 people must be born on same day of the week.

$\longrightarrow$ ~~30~~ $n = 30 \qquad m = 7$

$\therefore \quad \frac{n-1}{m} + 1 = 5$

$\therefore$ atleast 5 with same day of week

Q 7 colors to paints 50 bicycles. find out atleast how many bicycles will have same color

$\longrightarrow \quad \frac{49}{7} + 1 = 8$

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Q What will be minimum number of students in class so that atleast 3 born in same month.

$\longrightarrow$ $3 = \frac{n-1}{12} + 1$ $\therefore n-1 = 24$ $\therefore n = 25$

Q. $U = \{1, 2, 3, \ldots \ldots 10\}$

$A = \{2, 4, 6, 8, 10\}$ $B = \{1, 3, 5, 7, 9, 10\}$

i) $(A \cup B)' = \Phi$

ii) $(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

iii) $(B - A)' = \{2, 4, 6, 8, 10\}$

Q. A college gives info, 119 students enrolled in C.S. out of them 96 D.S., 53 foundation, 39 assembly language, 31 selected found. & assembly, 32 D.S. and assembly, 38 D.S. & foundation, 22 all 3.

$\longrightarrow$ Here $U = 119$

$A = 96$ $B = 53$ $C = 39$

$A \cap B = 38$ $B \cap C = 31$ $A \cap C = 32$

$A \cap B \cap C = 22$

By principle of inclusion & exclusion

$A \cup B \cup C = 96 + 53 + 39 - 38 - 31 - 32 + 22$

$= 109$

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