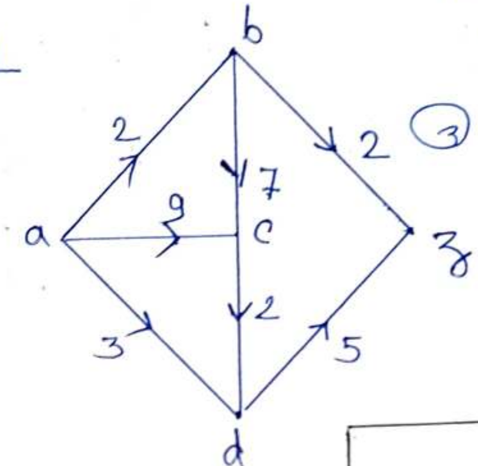
* Network Flow (Max Flow Min Cut)

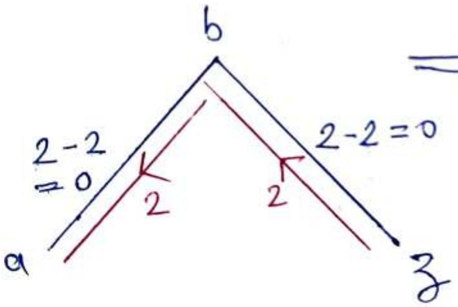
Ford Fulkerson Algorithm:-
1. Step with initial flow as 0
2. While there is a augmented path from source to sink
   - Add this path-flow to flow
3. Return flow

* Example: 1

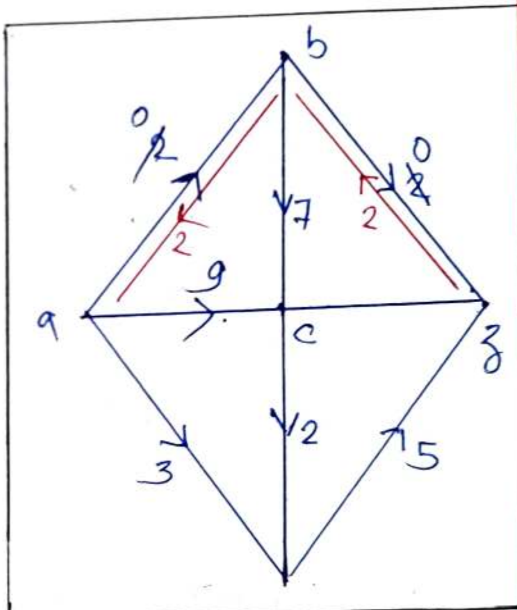


Solution:-

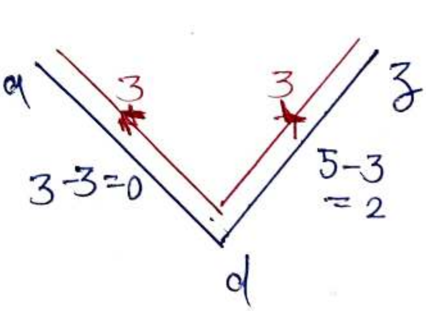
Step 1:→



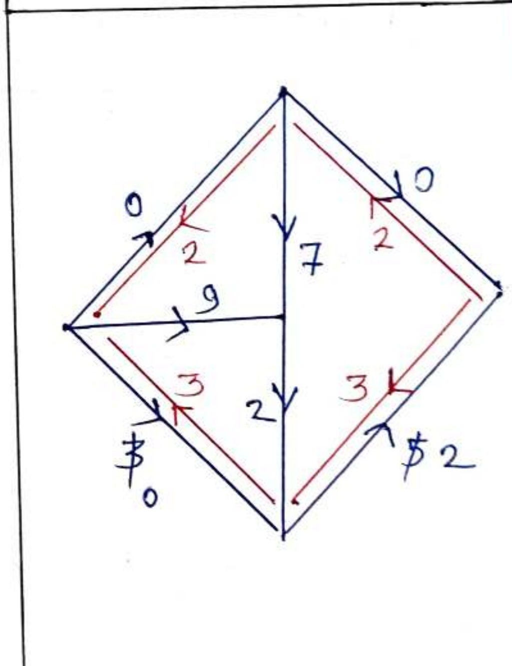
⟹



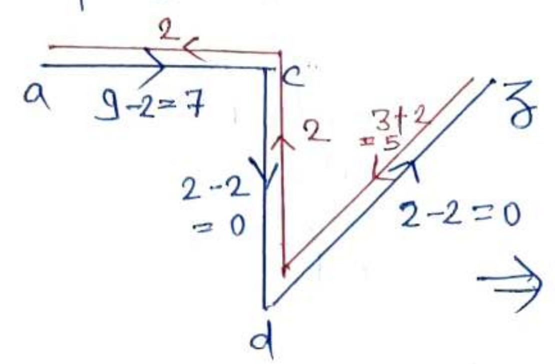
Step 2:→

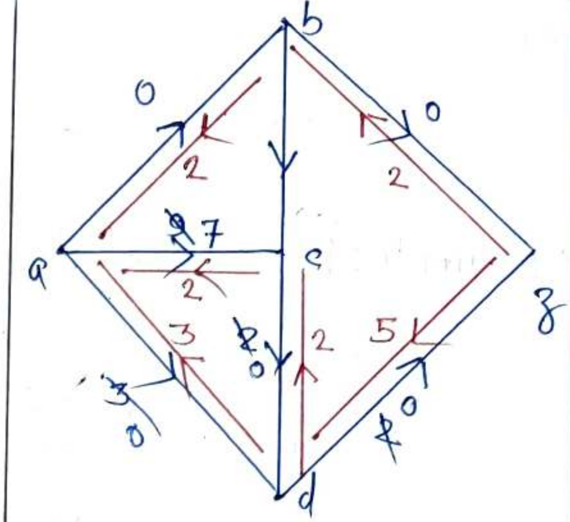


⟹

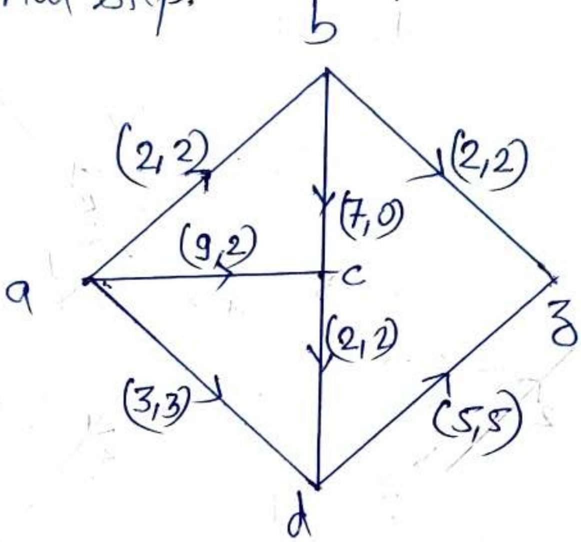


Step (3)



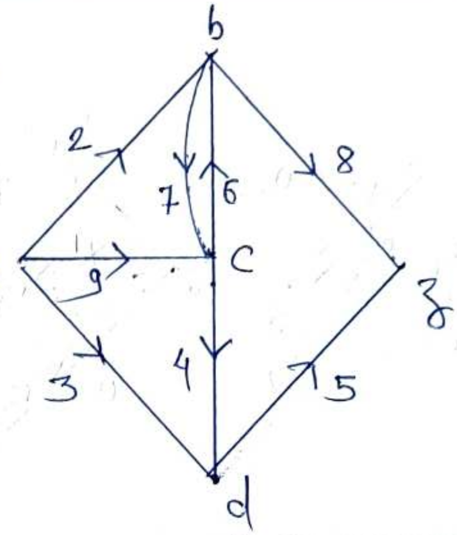


Step-(4) Final Step.



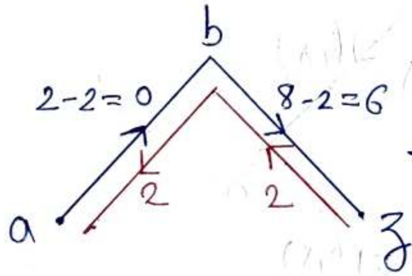
Maximal Flow = 5 + 2 = 7

Example 2:-

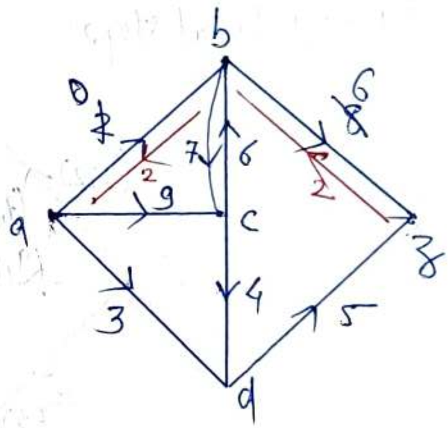


Solution:-

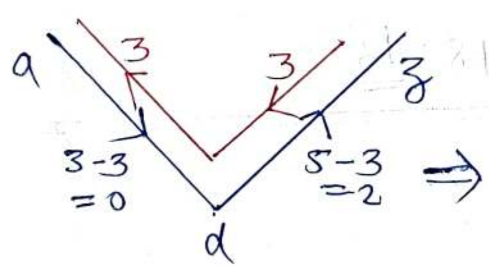
Step-1:-



⟹



Step-(2)



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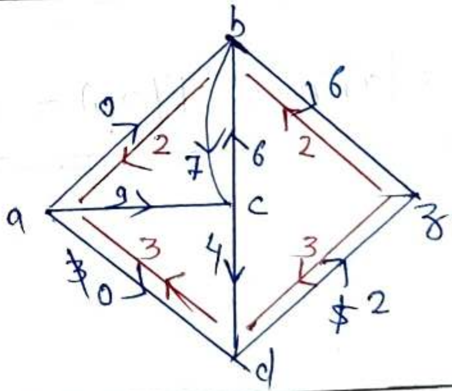
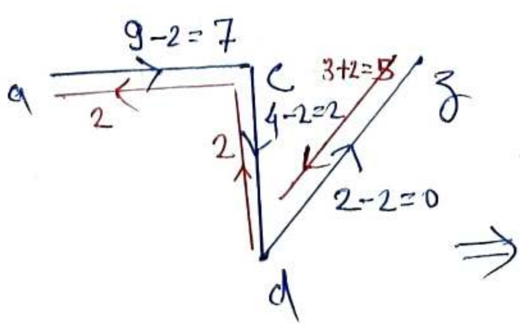
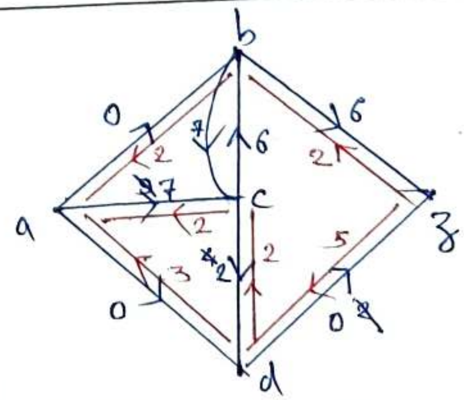


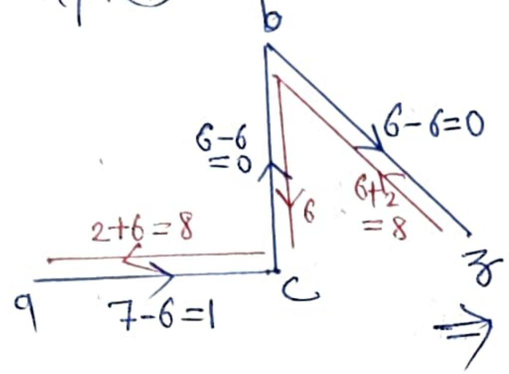
Step-(3)

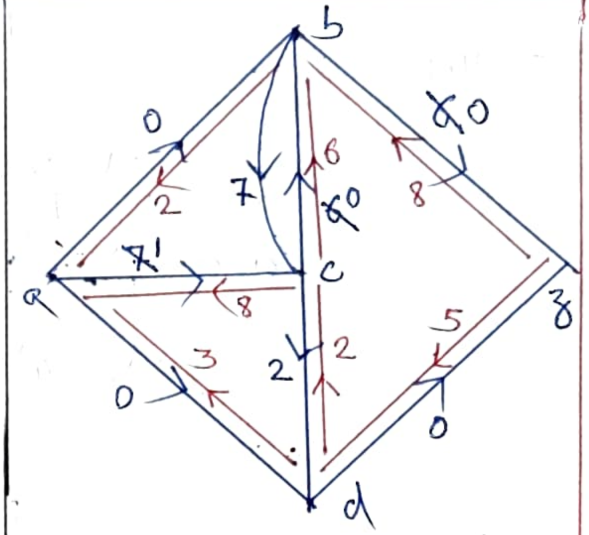


⟹

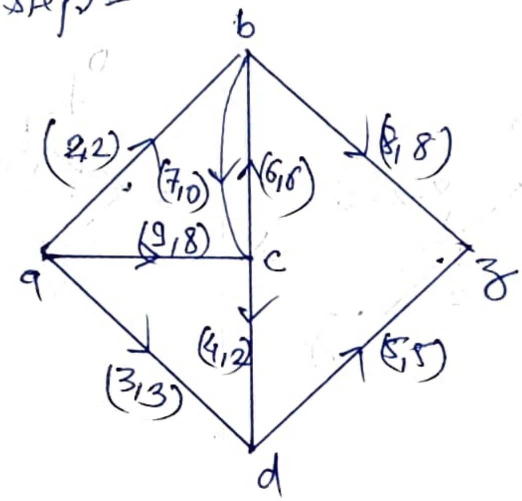


Step: 4



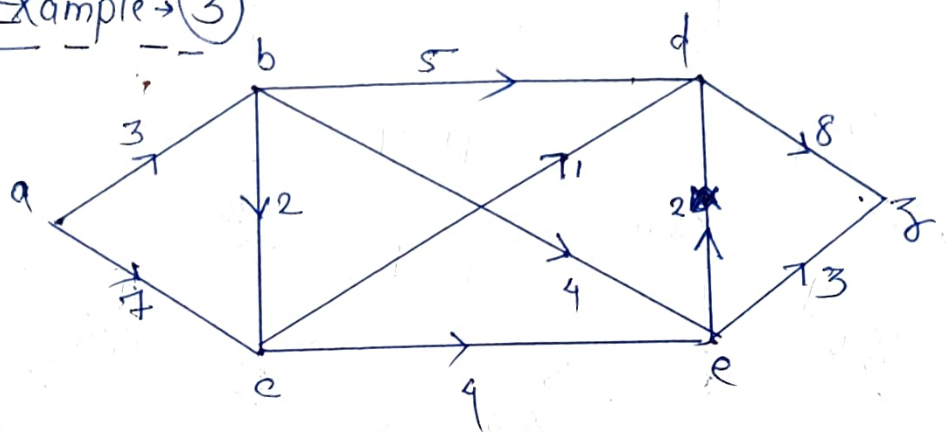


Step-5 Final step:-



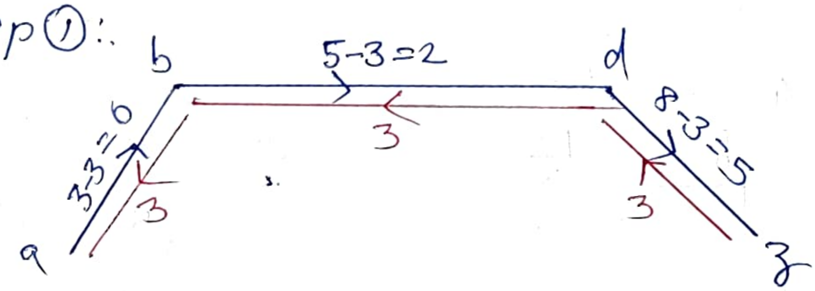
Maximal Flow = 5+8 = 13

Example → 3

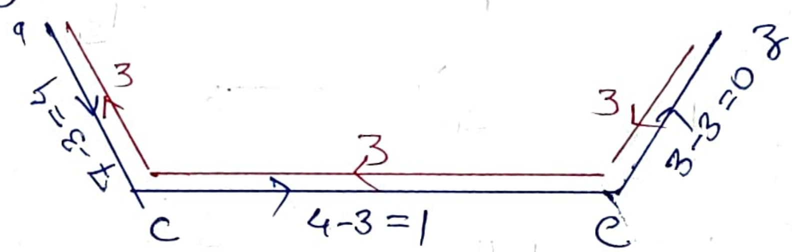


Solution

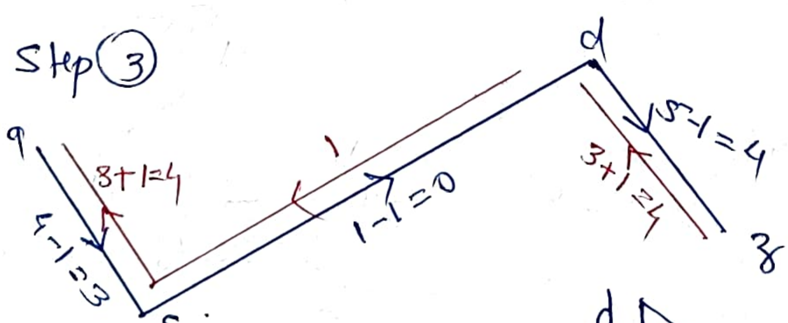
Step 1:



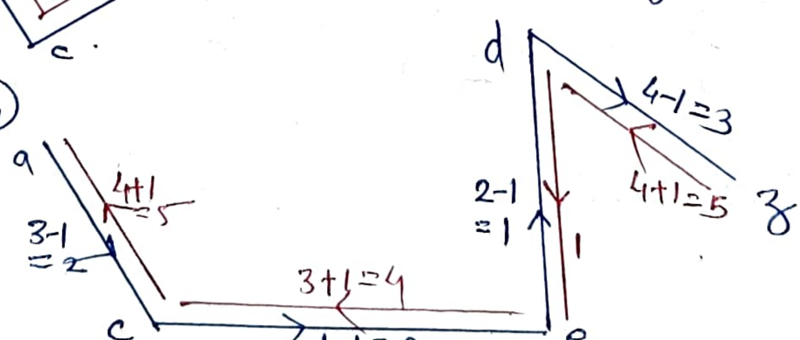
Step 2:

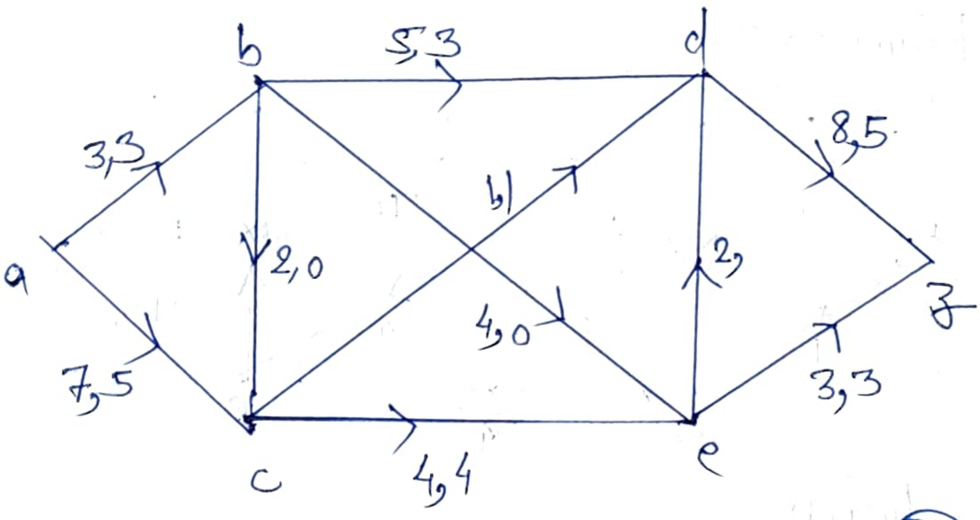


Step 3



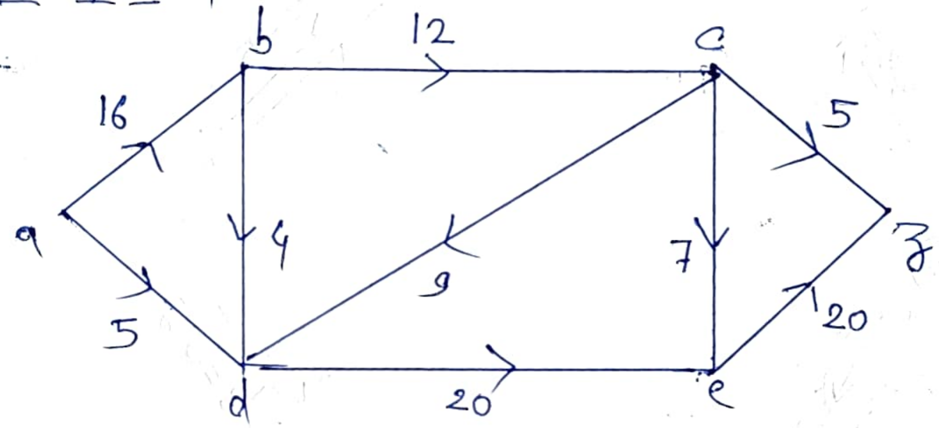
Step 4



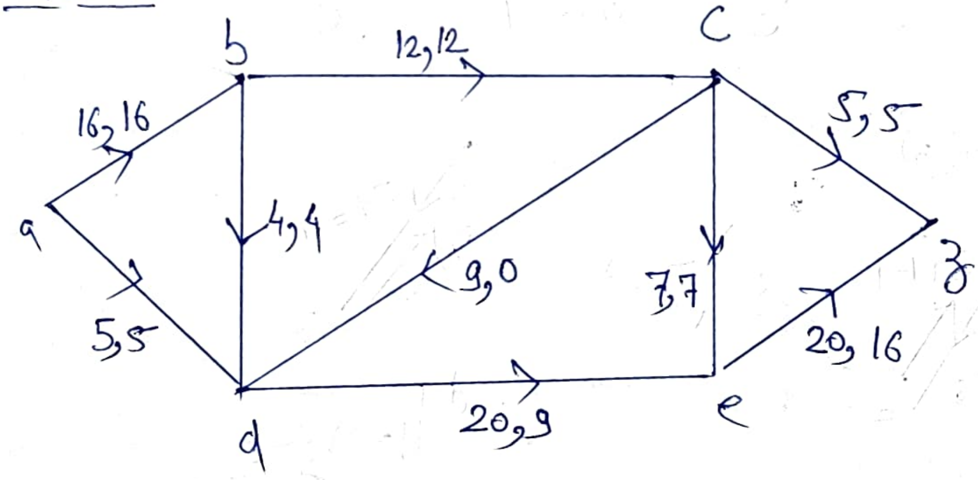


3+5=8

Example- 4



Solution

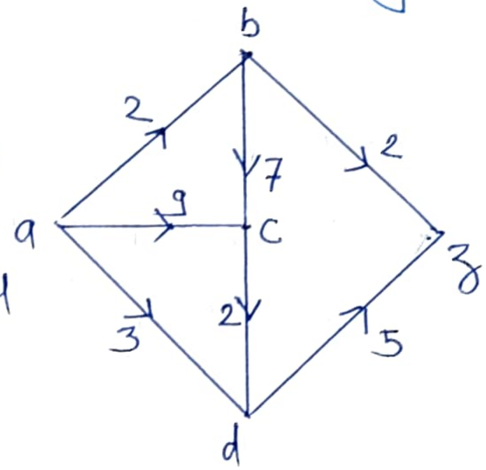


Max Flow = 21

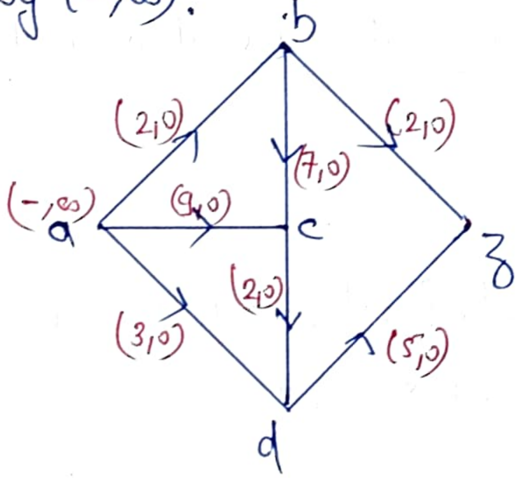
Page-(4)

* Labeling Procedure Algo

Example :- 1
Determine the maximal flow in the n/w using Labeling procedure algorithm.



Solution:-
step ①:- Assign the flow zero to each edge and label the source 'a' by $(-, \infty)$.



Step ②:-
Scan the vertices which are adjacent to the source 'a'. vertices b, c, and d are adjacent to 'a'.

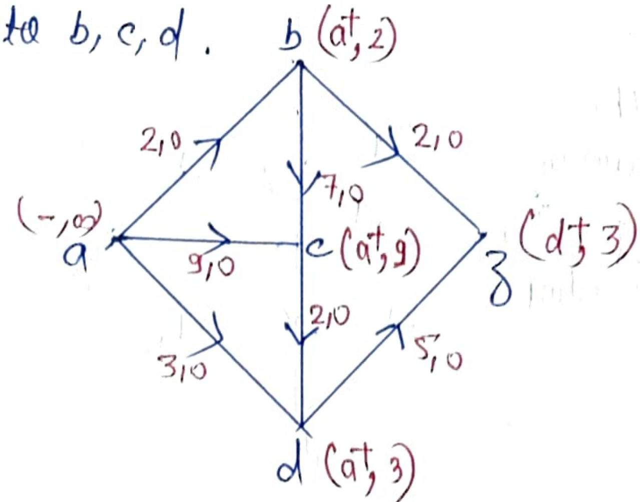
$b = (a^+, \Delta b)$, $\Delta b = c(a,b) - F(a,b)$
$= 2 - 0$
$\Delta b = 2$

$\therefore \underline{b = (a^+, 2)}$

$c = (a^+, \Delta c)$, $\Delta c = c(a,c) - F(a,c)$
$\Delta c = 9 - 0 = 9$
$\therefore \underline{c = (a^+, 9)}$

And $d = (a^+, \Delta d)$, $\Delta d = c(a,d) - F(a,d) = 3 - 0 = 3$
$\therefore \underline{d = (a^+, 3)}$

Assign label to b, c, d.



Similarly Calculate the value of z i.e. label to z (sink)
To find label of z, consider 'd' as a adjacent vertex of z
Note - (We can choose any vertex, b or d as both are adjacent to vertex z).

$\therefore z = (d^+, \Delta z)$

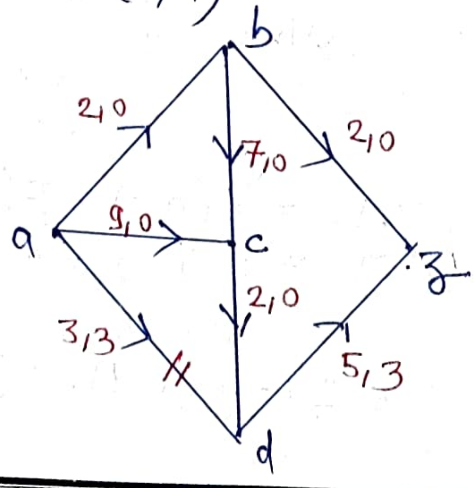
$\Delta z = \min(\Delta d, [C(d,z) - F(d,z)])$

$= \min(3, [5-0])$

$= \min(3, 5)$

$\underline{\Delta z = 3}$ $\qquad \therefore z = \underline{(d^+, 3)}$

Now,
According to label of the sink 'z', adjust the flow in the edge (d, z) and (a, d)



Repeat the step-2

Step-3 Select the edges (b,z) and (a,b) and find the new label of z with respect to b as adjacent vertex and adjust the flow from z to a.

$b = (a^+, 2)$

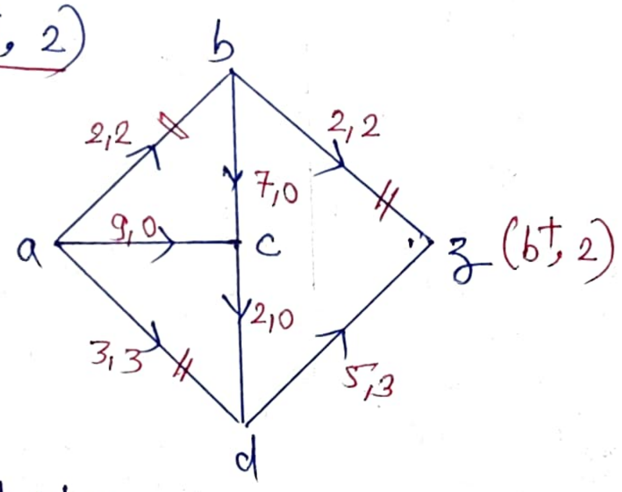
$z = (b^+, \Delta z)$

$\Delta z = \min(\Delta b, [C(b,z) - F(b,z)])$

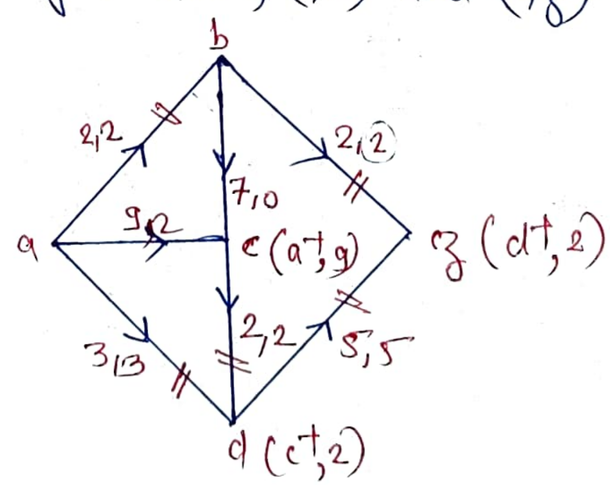
$= \min(2, (2-0))$

$\Delta z = 2$

$\therefore z = (b^+, 2)$

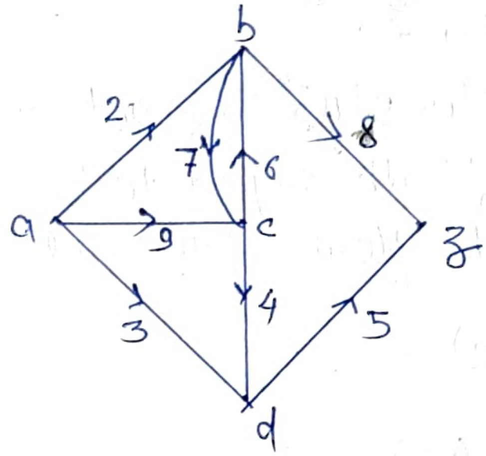


Step-4 Select edges, (a,c), (c,d) and (d,z)

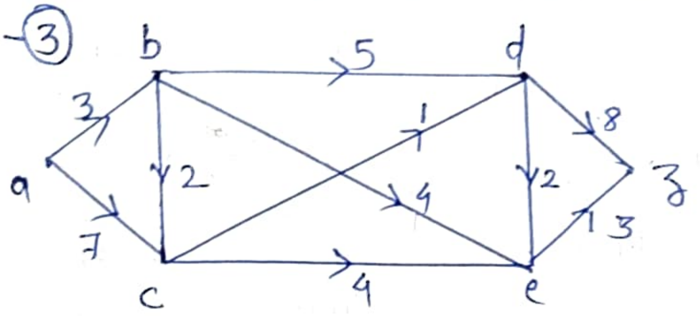


Maximal Flow = 7

Example - 2

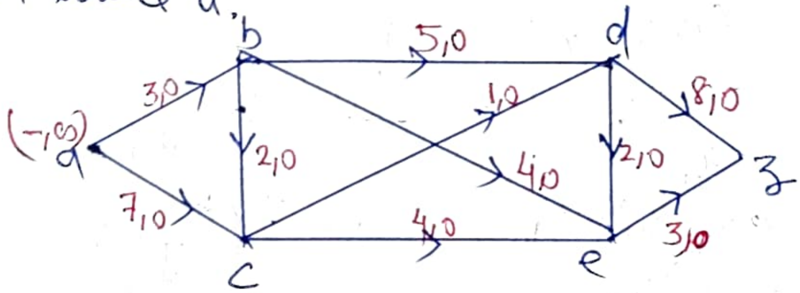


Example-③



Soln :-

Step-① Assign flow zero to each edge and label $(-,\infty)$ to the source a.



Label of b & c is,

$$b = (a^+, 3) \quad \& \quad c = (a^+, 7)$$

Vertex d and e are adjacent to b and c respectively
Therefore

$$d = (b^+, \Delta d) \quad \& \quad e = (c^+, \Delta e)$$

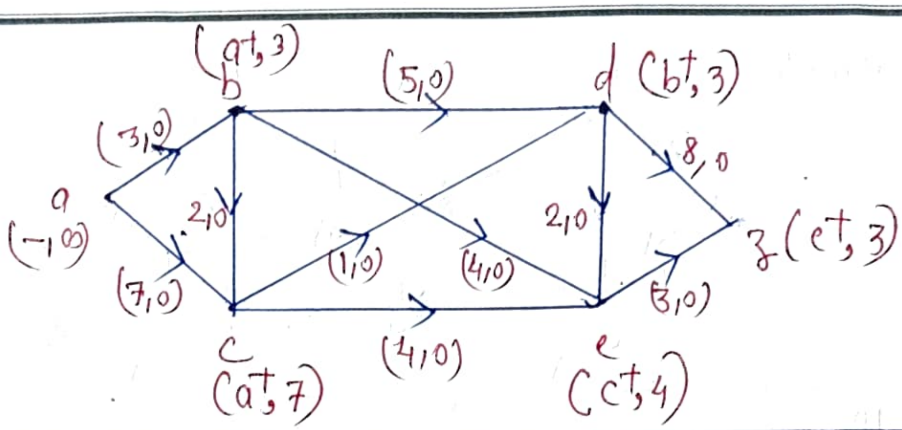
$$\Delta d = \min(\Delta b, C(b,d) - f(b,d)) = \min(3, 5-0)$$
$$\Delta d = 3$$

$$\Delta e = \min(\Delta c, [C(c,e) - f(c,e)]) = \min(7, 4-0)$$
$$\Delta e = 4$$

$$\therefore d = (b^+, 3) \quad \& \quad e = (c^+, 4)$$

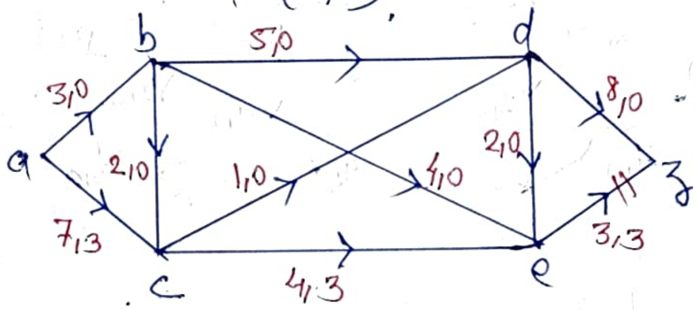
Simillarly

$z = (e^+, 3)$ — with respect e as adjacent vertex.

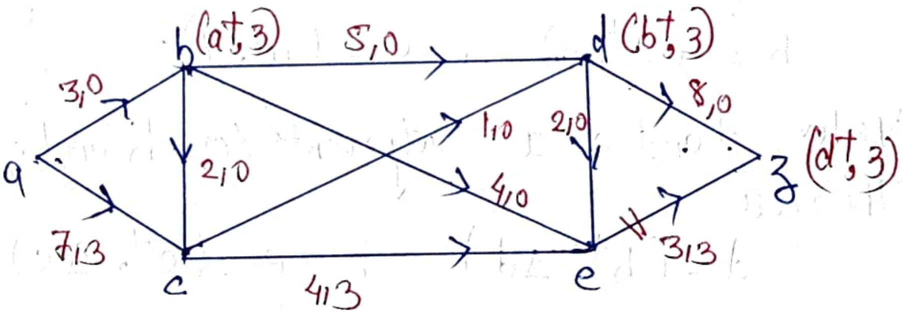


Step-2

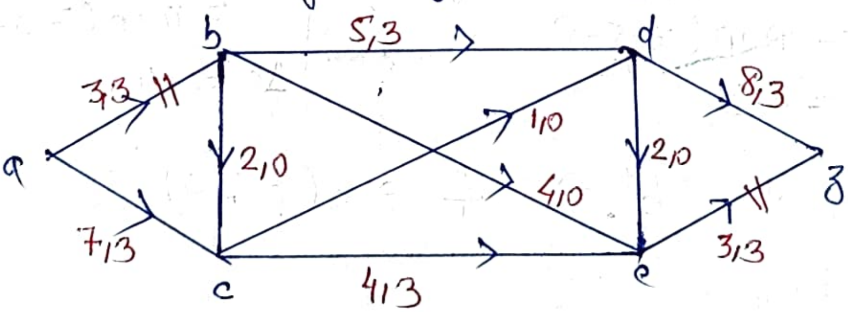
Acc to th label of sink z, adjust the flow in the edge (e,z) (c,e) and (a,c).



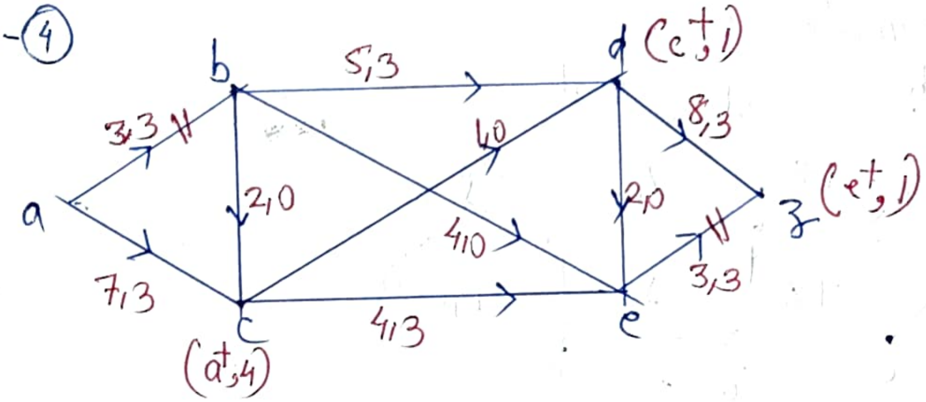
Step-3

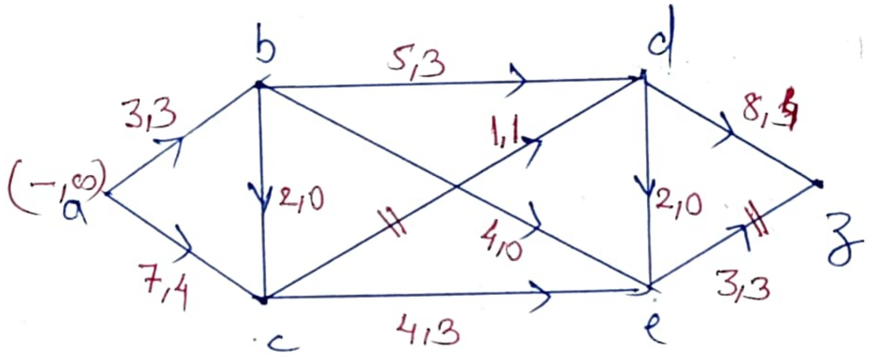


Adjust th flow to edge (d,z),(b,d) & (a,b)

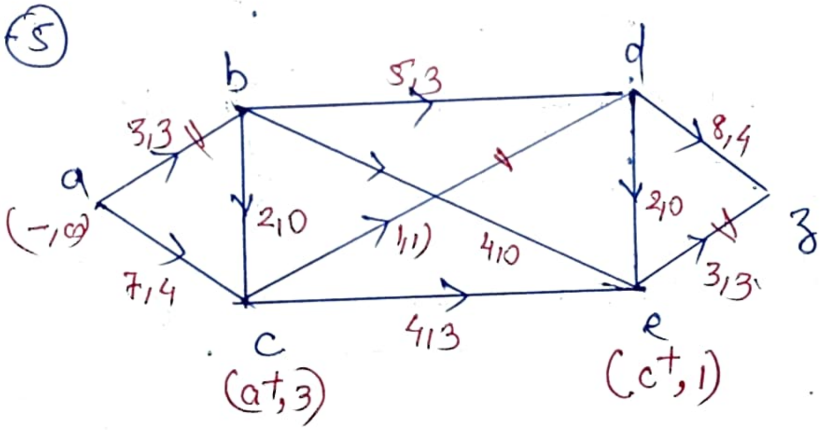


Step-4





Step 5



Since the vertex z (sink) cannot be labelled, we stop the procedure.

The vertex b, d, and z cannot be labelled because either edges approaching to vertices are saturated or the edges in the opposite direction has zero flow.

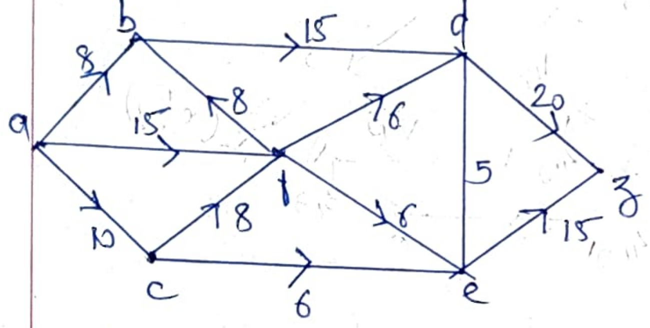
At this stage, minimum cut $(P, \bar{P})$ where

P is set of labelled vertex = (a, c, e) &

$\bar{P}$ is set of unlabelled vertex = (b, d, z)

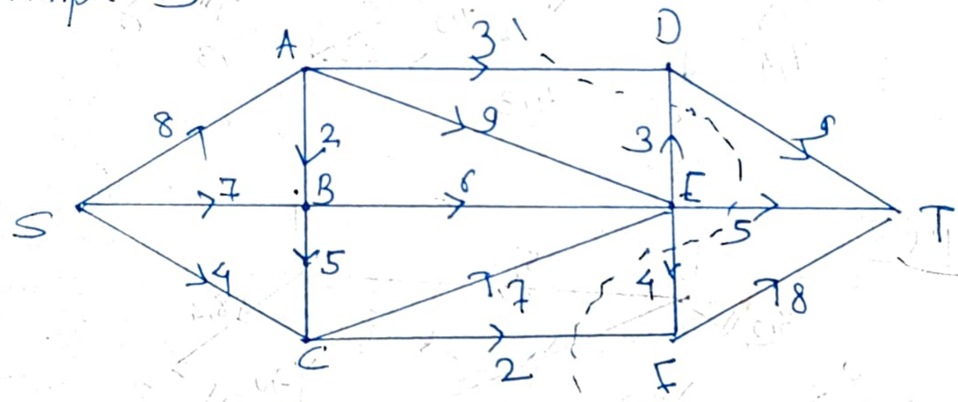
∴ Capacity of cut is $c(a,b) + c(c,d) + c(e,z) = 3+1+3 = 7$

Example - 4



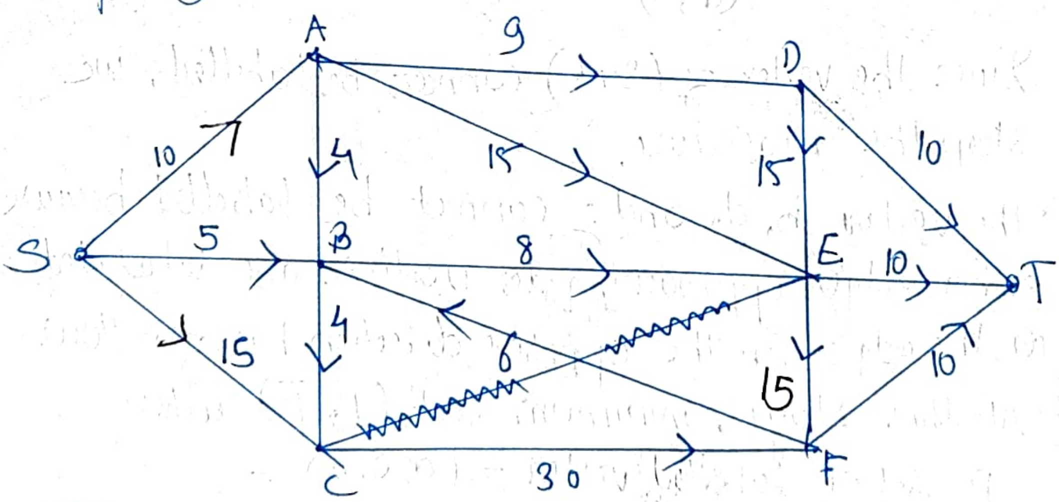
Solution → 21

Example - 5



Solution ⇒ 17

Example - 6



Solution — flow value - 28