Modern Education Society's Wadia College Of Engineering,Pune-01
Second Year Engineering
Academic Year : 2023-24
Assignment on Fourier Transform

1. Define the following:
   (a) Fourier Transform
   (b) Inverse Fourier Transform
   (c) Fourier Sine Transform
   (d) Inverse Fourier Sine Transform
   (e) Fourier Cosine Transform
   (f) Inverse Fourier Cosine Transform
2. Find the Fourier Transform of the following

(a) $f(x) = \begin{cases} \sin 2x & x \geq 0 \\ 0 & x < 0 \end{cases}$

Solution:

$$F(\lambda) = \int_{-\infty}^{\infty} f(x) e^{-i\lambda x} dx$$

$$F(\lambda) = \int_{0}^{\infty} \sin 2x e^{-i\lambda x} dx$$

Use

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx)$$

$$a = -i\lambda, b = 2$$

$$F(\lambda) = \left[\frac{e^{-i\lambda x}}{4 - \lambda^2}(-i\lambda \sin 2x - 2\cos 2x)\right]_0^{\infty}$$

$$F(\lambda) = 0 - \frac{1}{4 - \lambda^2}(0 - 2) = \frac{2}{4 - \lambda^2}$$

(b) $f(x) = \begin{cases} x^2 & x \geq 0 \\ 0 & x < 0 \end{cases}$

Solution:

$$F(\lambda) = \int_{-\infty}^{\infty} f(x)e^{-i\lambda x}dx$$

$$F(\lambda) = \int_{0}^{\infty} x^2 e^{-i\lambda x}dx$$

$$\int uvdx = uv_1 - u'v_2 + u''v_3 - \cdots$$

$$F(\lambda) = \left[x^2\frac{e^{-i\lambda x}}{-i\lambda} - 2x(\frac{e^{-i\lambda x}}{-\lambda^2}) + 2(\frac{e^{-i\lambda x}}{i\lambda^3})\right]_0^{\infty}$$

$$F(\lambda) = -\frac{2}{i\lambda^3} = \frac{2i}{\lambda^3}$$

(c) $f(x) = \begin{cases} x+5 & x \geq 0 \\ 0 & x < 0 \end{cases}$

Solution:

$$F(\lambda) = \int_{-\infty}^{\infty} f(x)e^{-i\lambda x}dx$$

$$F(\lambda) = \int_{0}^{\infty} (x+5)e^{-i\lambda x}dx$$

$$\int uvdx = uv_1 - u'v_2 + u''v_3 - \cdots$$

$$F(\lambda) = \left[(x+5)\frac{e^{-i\lambda x}}{-i\lambda} + (\frac{e^{-i\lambda x}}{\lambda^2})\right]_0^{\infty}$$

$$F(\lambda) = \frac{5}{-i\lambda} + \frac{1}{\lambda^2}$$

3. Find the Fourier Sine Transform of the following

(a) $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$

(b) $f(x) = \frac{e^{-ax}}{x}$

(c) $f(x) = e^{-2x} + e^{-3x}$

Solution: Hint:

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx)$$

$$F_s(\lambda) = \int_0^\infty f(x) \sin \lambda x dx$$

$$F_s(\lambda) = \int_0^\infty (e^{-2x} + e^{-3x}) \sin \lambda x dx$$

$$F_s(\lambda) = \int_0^\infty e^{-2x} \sin \lambda x dx + \int_0^\infty e^{-3x} \sin \lambda x dx$$

$$F(\lambda) = \left[\frac{e^{-2x}}{4 + \lambda^2}(-2 \sin \lambda x - \lambda \cos \lambda x) + \frac{e^{-3x}}{9 + \lambda^2}(-3 \sin \lambda x - \lambda \cos \lambda x) \right]_0^\infty$$

$$F(\lambda) = \frac{\lambda}{4 + \lambda^2} + \frac{\lambda}{9 + \lambda^2}$$

4. Find the Fourier Cosine Transform of the following

(a) $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$

(b) $f(x) = e^{|x|}$ for all x

(c) $f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$

(d) $f(x) = e^{-2x} + e^{-3x}$

Solution: Hint:

$$\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

5. Solve the Following Integral Equations :

   (a) $\int_0^\infty f(x)\sin\lambda x dx = \begin{cases} 1-\lambda & 0 \le \lambda \le 1 \\ 0 & \lambda \ge 1 \end{cases}$

   (b) $\int_0^\infty f(x)\sin\lambda x dx = \begin{cases} 1 & 0 \le \lambda \le 1 \\ 2 & 1 \le \lambda \le 2 \\ 0 & \lambda \ge 2 \end{cases}$

   (c) $\int_0^\infty f(x)\cos\lambda x dx = e^{-\lambda}, \lambda > 0$

   (d) $\int_0^\infty f(x)\cos\lambda x dx = \begin{cases} 1-\lambda & 0 \le \lambda \le 1 \\ 0 & \lambda \ge 1 \end{cases}$

6. Write short note on Discrete And Fast Fourier Transform

7. Write applications of Fourier Transform in different fields of Engineering.