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~~Homogeneous LDE~~

1. If the roots $m_1, m_2, m_3 \ldots m_n$ of auxiliary equation $\phi(D)=0$ are real and distinct, then solution of $\phi(D)y=0$ is distinct, then solution of

Solution: A.) $c_1 e^{m_1 x} + c_2 e^{m_2 x} + \ldots + c_n e^{m_n x}$

2. The solution of differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ is,

Solution: let $D = \frac{d}{dx}$

$$\therefore D^2y - 5Dy + 6y = 0$$

$$\therefore D^2 - 5D + 6 = 0 \quad \text{is the AE}$$

$$(D-3)(D-2) = 0$$

$$\therefore D = 3, 2$$

if $D = a_1, a_2$ then

$$CF = C_1 e^{a_1 x} + C_2 e^{a_2 x}$$

$$= C_1 e^{2x} + C_2 e^{3x}$$

A) $C_1 e^{2x} + C_2 e^{3x}$

3. The solution of differential equation $\frac{d^2y}{dx^2} - 4y = 0$ is,

Solution: let $D = \frac{d}{dx}$

$$D^2y - 4y = 0$$

$$\therefore D^2 - 4 = 0$$

$$D = \pm 2$$

if $D = a_1, a_2$

then $CF = C_1 e^{a_1 x} + C_2 e^{a_2 x}$

$$\therefore CF = C_1 e^{2x} + C_2 e^{-2x}$$

D) $C_1 e^{-2x} + C_2 e^{2x}$

4\. The solution of differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ is,

Soln: Let $D = \frac{d}{dx}$

$$D^2y - Dy - 2y = 0$$
$$(D^2 - D - 2)y = 0$$
$\therefore D^2 - D - 2 = 0$ is the AE
$$D^2 - 2D + D - 2 = 0$$
$$D(D-2) + 1(D-2) = 0$$
$$(D+1)(D-2) = 0$$
$$\therefore D = -1, 2$$
if $D = a_1, a_2$
then $CF = C_1e^{a_1x} + C_2e^{a_2x}$
$$CF = C_1e^{-x} + C_2e^{2x}$$

c) $C_1e^{2x} + C_2e^{-x}$

5\. The solution of differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ is,

Soln: Let $D = \frac{d}{dx}$

$$D^2y + 2Dy + y = 0$$
$(D^2 + 2D + 1) = 0$ is the AE
$$(D+1)^2 = 0$$
$$D = -1, -1$$
if $D = a, a$ then
$$CF = [C_1 + C_2x]e^{ax}$$
$$= [C_1 + C_2x]e^{-x}$$

c) $[C_1 + C_2x]e^{-x}$

6. Particular integral of linear differential equation with constant coefficient $\phi(D)y = f(x)$

Soln: A) $\frac{1}{\phi(D)} f(x)$

7. Particular integral $\frac{1}{D+1} e^{e^x}$ where $D = \frac{d}{dx}$ is,

Soln:

$$y_p = \frac{1}{D+1} e^{e^x}$$

$$= e^{-x} \int e^x e^{e^x} dx$$

Let $e^x = t$

$e^x dx = dt$

$$= t^{-1} \int e^t dt$$

$$= t^{-1} e^t$$

$$y_p = e^{-x} e^{e^x}$$

A) $e^{-x} e^{e^x}$

8. Particular integral $\frac{1}{D+1} \sin e^x$, where $D = \frac{d}{dx}$ is,

Soln:

$$y_p = \frac{1}{D+1} \sin e^x$$

$$= e^{-x} \int e^x \sin e^x dx$$

let $e^x = t$

$e^x dx = dt$

$$= t^{-1} \int \sin t \, dt$$

$$= -t^{-1} \cos t$$

$$= -e^{-x} \cos e^x$$

D) $-e^{-x} \cos e^x$

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9. particular integral $(D^2+4D+3)y = e^{-3x}$, where $D = \frac{d}{dx}$ is

Soln: $y_p = \frac{1}{\phi(D)} f(x)$

$= \frac{1}{D^2+4D+3} e^{-3x}$

$D \to -3$

$= \frac{1}{9-12+3}$

$= \frac{1}{0} e^{-3x}$

(case of failure)

$= x\left[\frac{1}{2D+4} e^{-3x}\right]$

$D \to -3$

$y_p = \frac{x e^{-3x}}{-2}$

c) $-\frac{x}{2} e^{-3x}$

10. Particular integral $(D^2-5D+6)y = 3e^{5x}$ is

Soln: $y_p = \frac{1}{\phi(D)} f(x)$

$= \frac{1}{D^2-5D+6} 3e^{5x}$

$D \to 5$

$= \frac{1}{25-25+\cancel{6}2} \times \cancel{3}^{1} e^{5x}$

$y_p = \frac{e^{5x}}{2}$

A) $\frac{e^{5x}}{2}$

11. To reduce the differential equation $(x+2)^2 \frac{d^2y}{dx^2} - 4(x+2)\frac{dy}{dx} + 6y = 4x+7$ to linear differential equation with constant coefficients substitution is,

Soln: c) $x+2 = e^z$

12. Solution of differential equations $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = x$ is

Soln: ~~put $x = e^z$~~ multiply by x,

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = x^2$$

$$x\frac{dy}{dx} = Dy$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

put $x = e^z$

$z = \log x$

$$D(D-1)y + Dy = e^{2z}$$

$$D^2y = e^{2z}$$

$\therefore D^2 = 0$ is the AE

$D = 0, 0$

if $D = a, a$

then $CF = [C_1 + C_2 z]e^{az}$

$= [C_1 + C_2 z]$

$$y_p = \frac{1}{\phi(D)} f(z)$$

$$= \frac{1}{D^2} e^{2z}$$

$D \to 2$

$$= \frac{e^{2z}}{4}$$

∴ General solution,

$y = y_c + y_p$

$= C_1 + C_2 \cancel{z} + \frac{e^{2z}}{4}$

$= C_1 + C_2 \log x + \frac{x^2}{4}$

D) $[C_1 \log x + C_2] + \frac{x^2}{4}$

13. The differential equation

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = \frac{x^2}{1+x^2}$, on putting

$x = e^z$ and using $D = \frac{d}{dz}$ is transformed into

Soln:

put $x = e^z$

$x^2 \frac{d^2y}{dx^2} \cancel{+ x\frac{dy}{dx}} = D(D-1)y$

$x\frac{dy}{dx} = Dy$

put $x = e^z$

$z = \log x$

$D(D-1)y + Dy - y = \frac{e^{3z}}{1+e^{3z}}$

$D^2y - y = \frac{e^{2z}}{1+e^{2z}}$

$(D^2-1)y = \frac{e^{2z}}{1+e^{2z}}$

∴ $D^2 - 1 = 0$ is the AE

$D = \pm 1$

if $D = a_1, a_2$

then $CF = C_1 e^{a_1 x} + C_2 e^{a_2 x}$

$= C_1 e^x + C_2 e^{-x}$

~~Ans~~ c) $x+y = C_1$, $y - z = C_2$