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UNIT: NO: 2: (TWO) :-

FOURIER AND Z - TRANSFORM.

(1.) Simultaneous LDE.

(2.) Symmetrical LDE.

Ex. $\frac{dx}{dt} + 2y + x = \sin t$

$$\frac{dy}{dt} + 2\cdot\frac{dx}{dt} + y = 0.$$

- Exeersize 2.1.

① SOLVE :- $\frac{dx}{dt} + y = e^{t}$.

$$\frac{dy}{dt} - x = e^{-t}$$

ANS. SOLUTION :- Denote $D = \frac{d}{dt}$

$\therefore \quad Dx + y = e^{t} \longrightarrow ①$

$-x + Dy = e^{-t} \longrightarrow ②$

Multiply Eqn ② by D ; we get.

$\therefore \quad -Dx + D^2y = D(e^{-t}) = -e^{-t} \longrightarrow ③$

Add Eqn ① and ③

$$\cancel{Dx} + y = e^{t}$$
$$-\cancel{Dx} + D^2y = -e^{-t}$$
$$(D^2+1)\,y = e^{t} - e^{-t} \longrightarrow ④$$

A.E. : $D^2+1 = 0$

$\therefore \quad D = \pm i$

$$\boxed{y_c = c_1\cos t + c_2 \sin t}$$

$$y_p = \frac{1}{D^2+1}\cdot e^{-t} - \frac{1}{D^2+1}\cdot e^{-t} = \frac{1}{2}\left[e^{t} - e^{-t}\right]$$

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$\therefore y = y_c + y_p$

$$\therefore \boxed{y = c_1 \cdot \cos t + c_2 \sin t + \frac{1}{2}\left[e^{t} - e^{-t}\right]}$$

From Equation (2)

$$x = \frac{dy}{dt} - e^{-t}$$

(2) Solve:- $\frac{dx}{dt} + 5x - 2y = t.$

$$\frac{dy}{dt} + 2x + y = 0.$$

ANS. SOLUTION:- Denote, $D = d/dt.$

$\therefore \quad Dx + 5x - 2y = t.$

$-\ (D+5)x - 2y = t \longrightarrow (1)$

$\therefore \quad Dy + 2x + y = 0$

$-\ (D+1)y + 2x = 0 \longrightarrow (2)$

$(D+1) \times$ Eqn (1) $+ 2 \times$ Eqn (2)

$$(D+1)(D+5)x - 2\cancel{(D+1)}y = (D+1)t = 1+t$$

$$4x + 2\cancel{(D+1)}y = 0$$

$$(D^2+6D+9)x = 1+t.$$

A.E. : $D^2+6D+9 = 0$

$\therefore \quad D = -3, -3$

$$x_c = (c_1 t + c_2) \cdot e^{-3t} \longrightarrow (1)$$

$$\therefore x_p = \underset{(D=0)}{\frac{1}{D^2+6D+9}} \times (1) + \frac{1}{D^2+6D+9} \times t$$

$$\therefore x_p = \frac{1}{9} + \frac{1}{9\left[\left(1+\left(\frac{D^2+6D}{9}\right)\right)\right]} \times t.$$

$$x_p = \frac{1}{9} + \frac{1}{9}\left[1+\left(\frac{D^2+6D}{9}\right)\right]^{-1} x t$$

$$x_p = \frac{1}{9} + \frac{1}{9}\left[1-\left(\frac{D^2+6D}{9}\right)\right] \cdot t$$

$$\boxed{x_p = \frac{1}{9} + \frac{1}{9}\left[t - \frac{6}{9}\right]} \longrightarrow \text{(ii)}$$

$$x = x_c + x_p$$

$$x = (c_1 t + c_2) \cdot e^{-3t} + \frac{1}{9} + \frac{1}{9}\left[t - \frac{6}{9}\right]$$

$$y = \frac{1}{2}\left[\frac{dx}{dt} + 5x - t\right]$$