Unit 5 Numerical Methods :-

1) Sol^n of algebraic or transcendental eq^n
- Bisection Method
- False position
- Secant method
- Newton Raphson Method

2) Sol^n of simultaneous equations
↳ Gauss elimination
Gauss – Jordon Method
LU decomposition method
Jacobi iteration method
Gauss - Seidal iterative method.

1) Gauss Elimination :- $AX = B$

$A \rightarrow$ Upper triangular form by using row transformations.

2) Gauss - Jordon : $AX = B$

$A \longrightarrow$ Diagonal Matrix by using row transformations.

3) LU-decomposition $AX = B$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$L\underline{UX} = B$

denote $UX = V$ ——— ①

$\rightarrow$ $LV = B$. $\longrightarrow$ We find value of V

Substitute in eq^n ①, we get value of X.

LU-decomposition method :-

Ex :- Solve by Gauss-elimination method, the system of equations:

$$2x_1+x_2+x_3=10$$
$$3x_1+2x_2+3x_3=18$$
$$x_1+4x_2+9x_3=16$$

$$\boxed{\begin{matrix} x=7 \\ y=-9 \\ z=5 \end{matrix}}$$

$\rightarrow$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

R_{13}

$$\begin{bmatrix} 1 & 4 & 9 \\ 3 & 2 & 3 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 16 \\ 18 \\ 10 \end{bmatrix}$$

$18-3\times 16$

$10-32$

$R_2-3R_1,$
R_3-2R_1

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & -10 & -24 \\ 0 & -7 & -17 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 16 \\ -30 \\ -22 \end{bmatrix}$$

$\dfrac{R_2}{-2}$

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & 5 & 12 \\ 0 & -7 & -17 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 16 \\ 15 \\ -22 \end{bmatrix}$$

$-17\times 5+7\times 12$

$5\times -22+7\times 15$

$5R_3+7R_2$

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & 5 & 12 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 16 \\ 15 \\ -5 \end{bmatrix}$$

$x+4y+9z=16 \Rightarrow x=7$

$5y+12z=15 \Rightarrow y=-9$

$-z=-5 \Rightarrow \boxed{z=+5}$

Use LU decomposition method, solve

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

Sol^n:-

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

$u_{11} = 2$, $u_{12} = 1$, $u_{13} = 4$

$l_{21}u_{11} = 8 \Rightarrow l_{21} = 8/2 = 4$

$l_{21}u_{12} + u_{22} = -3 \Rightarrow u_{22} = -7$

$l_{21}u_{13} + u_{23} = 2 \Rightarrow u_{23} = -14$

$l_{31}u_{11} = 4 \Rightarrow l_{31} = 2$

$l_{31}u_{12} + l_{32}u_{22} = 11 \Rightarrow l_{32} = -9/7$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = -1 \Rightarrow u_{33} = -27$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -9/7 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -9/7 & 1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 20 \\ 33 \end{pmatrix}$$

$$v_1 = 12$$

$$4v_1 + v_2 = 20 \quad \Rightarrow \quad v_2 = -28$$

$$2v_1 - \frac{9}{7} v_2 + v_3 = 33 \quad \Rightarrow \quad v_3 = 4$$

$$\therefore \begin{pmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ -28 \\ 4 \end{pmatrix}$$

$$\left.\begin{aligned} -27z &= 4 \\ -7y - 14z &= -28 \\ 2x + y + 4z &= 12 \end{aligned}\right\} \Rightarrow \begin{aligned} z &= 4/-27 \\ y &= \\ x &= \end{aligned}$$