Tutorial No. 11

Q1] Solve the following.

1.) find lagrange's interpolating polynomial passing through set of points.

x	0	1	2
y	4	3	6

use it to find y at $x = 1.5$, $\frac{dy}{dx}$ at $x = 0.5$ and find $\int_0^3 y \cdot dx$.

→ $y = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$ — ①

$$L_0(x) = \frac{(x-1)(x-2)}{(0-1)(0-2)} = \frac{1}{2}(x^2-3x+2)$$

$$L_1(x) = \frac{(x-0)(x-2)}{(1-0)(1-2)} = -(x^2-2x)$$

$$L_2(x) = \frac{(x-0)(x-1)}{(2-0)(2-1)} = \frac{1}{2}(x^2-x)$$

given $x_0 = 0$ $y_0 = 4$ $x_1 = 1$ $x_2 = 2$ $y_1 = 3$ $y_2 = 6$

$$y = \frac{4}{2}\cdot(x^2-3x+2) + 3(-(x^2-2x)) + \frac{6}{2}(x^2-x)$$

$$y = 2x^2 - 3x + 4$$

$$y_{x=1.5} = 2(1.5)^2 - 3(1.5) + 4 = 4.$$

$$\frac{dy}{dx} = 4x-3\Big|_{x=0.5} = 4\times(0.5) - 3 = -1$$

$$\int_0^3 y \cdot dx = \int_0^3 2x^2 - 3x + 4 \; dx = \left[2\cdot\frac{x^3}{3} - \frac{3x^2}{2} + 4x\right]_0^3$$

$$= 16.5$$

2.) Construct Newtons forward interpolation polynomial for following data

x	4	6	8	10
y	1	3	8	16

∴ evaluate y for $x=5$.

→

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
4	1 y_0	2 Δy_0	3	
6	3	5	3 $\Delta^2 y_0$	0
8	8	8		
10	16			

$$y = f(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2u}\,\Delta^2 y_0 + \cdots$$

$$u = \frac{x - x_0}{h} = \frac{x-4}{2}$$

$$\therefore y = 1 + \left(\frac{x-4}{2}\right)\times 2 + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}-1\right)}{2}\times 3$$

$$= 1 + (x-4) + \left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)\times\frac{3}{2}$$

$$= 1 + (x-4) + \frac{3}{8}(x^2 - 10x + 24)$$

$$= \frac{3}{8}x^2 - \frac{30}{8}x + 9 + x - 4 + 1 = \frac{3x^2}{8} - \frac{22x}{8} + 6$$

$$y(5) = 1.625$$

Q2.] Numerical Integration.

1.) Use Trapezoidal rule to numerically evaluate $\int_0^{0.6} e^{-x^2} dx$. by tabling 7 coordinates.

→ $y = e^{-x^2}$

0 .1 .2 .3 .4 .5 .6

x	0	0.1	0.2	0.3	0.4	0.5	0.6
y	1	0.9900	0.9608	0.9139	0.8521	0.7788	0.6977

$$\int_0^{0.6} e^{-x^2} dx = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{0.1}{2}\left[(1+0.6977) + 2(0.99 + 0.9608 + 0.9139 + 0.8521 + 0.7788)\right]$$

$$= 0.53445$$

2.) Use simpsons $\frac{1}{3}$rd rule to obtain $\int_0^{\pi/2} \frac{\sin x}{x} dx$ by dividing interval into four parts.

→ $h = \frac{\pi}{8}$

using $\lim_{x\to 0} \frac{\sin x}{x} = 1$, value of x & $y = \frac{\sin x}{x}$

x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
y	1.0 y_0	0.9744953 y_1	0.9003163 y_2	0.7842133 y_3	0.6366197 y_4

$$I = h/3\left[(y_0+y_4) + 4(y_1+y_3) + 2y_2\right]$$

$$= \pi/24\left[1 + 0.6366197 + 4(0.9744953 + 0.7842133) + 2(0.9003163)\right]$$

$$= 1.3707929$$

3.) Evaluate $\int_0^{\pi} \frac{\sin^2\theta}{5+4\cos\theta} d\theta$, By simpson's $\frac{3}{8}^{th}$ rule. $h = \frac{\pi}{6}$

→ $y = \frac{\sin^2\theta}{5+4\cos\theta}$

θ	0	π/6	π/3	π/2	2π/3	5π/6	π
y	0 y_0	0.0295365 y_1	0.1071428 y_2	0.2 y_3	0.25 y_4	0.1622711 y_5	0.0 y_6

$I = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_4) + 3(y_2+y_5) + 2y_3\right]$

$= \frac{3}{8} \times \frac{\pi}{6} \left[(0+0) + 3(0.0295365+0.25) + 3(0.1071428+0.1622711) + 2(0.2)\right]$

$= \frac{\pi}{16}[2.0483512]$

$= 0.4021928$

Q3] Solve the following.

1.) Use Euler's method to solve the equation
$\frac{dy}{dx} = 1+xy$
subject to conditions at $x=0, y=1$ & tabulate y for $x = 0\ (0.1)\ (0.5)$

$f(x,y) = 1+xy$
$h = 0.1$
$x_0 = 0$
$y_0 = 1$

① $f(x_0, y_0) = 1 + x_0 y_0 = 1 + (0)(0) = 1$

$y_1 = y_0 + h f(x_0, y_0)$
$= 1 + (0.1)(1)$
$= 1.1$

② $f(x_1 y_1) = 1 + x_1 y_1 = 1 + (0.1)(1.11) = 1.11$

$y_2 = y_1 + h f(x_1, y_1)$
$= 1.1 + (0.1)(1.11)$
$= 1.211$

③ $f(x_2 y_2) = 1 + x_2 y_2 = 1 + (0.2)(1.211) = 1.2422$

$y_3 = y_2 + h f(x_2 y_2)$
$= 1.211 + (0.1)(1.242)$
$= 1.33522$

④ $f(x_3 y_3) = 1 + x_3 y_3 = 1 + (0.3)(1.3352) = 1.40056$

$y_4 = y_3 + h f(x_3 y_3)$
$= 1.3352 + (0.1)(1.40056)$
$= 1.475246$

⑤ $f(x_4, y_4) = 1 + x_4 y_4 = 1 + (0.4)(1.4753) = 1.59012$

$y_5 = y_4 + h f(x_4 y_4)$
$= 1.475246 + (0.1)(1.59012)$
$= 1.634312$

Tabulated solution is.

x	0	0.1	0.2	0.3	0.4	0.5
y	1	1.1	1.211	1.3352	1.4753	1.6343