TUTORIAL – 2 (TWO) :-

Ex 1] SOLVE :- $\frac{d^2y}{dx^2} + 3 \cdot \frac{dy}{dx} + 2y = e^{e^x}$

ANS. SOLUTION :-

The Auxiliary Equation (A.E.) of above given equation is :-

$$D^2 + 3D + 2 = 0$$

$$\therefore (D+2)(D+1) = 0$$

$$\boxed{D = -1, -2}$$

Thus, $C.F. = y_c = c_1 \cdot e^{-x} + c_2 \cdot e^{-2x}$

$$\boxed{y_c = c_1 \cdot e^{-x} + c_2 \cdot e^{-2x}} \longrightarrow (1)$$

Here, Particular Integral (P.I.)

$$P.I. = y_p = \frac{1}{(D+2)(D+1)} \cdot e^{e^x}$$

$$\therefore P.I. = \frac{1}{(D+2)} \left[\frac{1}{(D+1)} \cdot e^{e^x} \right]$$

$$\therefore P.I. = \frac{1}{(D+2)} \left[e^{-x} \int e^x \cdot e^{e^x} \cdot dx \right]$$

Put, $e^x = t \Rightarrow e^x \, dx = dt$.

$$\therefore P.I. = \frac{1}{(D+2)} \left[e^{-x} \int e^t dt \right] = \frac{1}{(D+2)} \left[e^{-x} \, e^t \right]$$

$$\therefore P.I. = \frac{1}{(D+2)} \left[e^{-x} \cdot e^{e^x} \right]$$

$$\therefore P.I. = e^{-2x} \int e^{+2x} \cdot e^{-x} \cdot e^{e^x} dx$$

$$\therefore P.I. = e^{-2x} \int e^x \cdot e^{e^x} dx$$

Again put $e^x = t \Rightarrow e^x dx = dt$.

$$P.I. = e^{-2x} \int e^t dt = e^{-2x} \cdot e^t = e^{-2x} \cdot e^{e^x}$$

$$\boxed{P.I. = y_p = e^{-2x} \cdot e^{e^x}} \longrightarrow (2)$$

Hence, the Complete Solution will be :-

$$y = y_c + y_p$$

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$$\rightarrow \boxed{y = c_1 \cdot e^{-x} + c_2 \cdot e^{-2x} + e^{-2x} \cdot e^{x}}$$

Ex 2] **SOLVE:-** $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = \dfrac{1}{1+e^x}$

ANS. **SOLUTION:-** We know, $D = \dfrac{d}{dx}$

$$\therefore (D^2+D)y = \frac{1}{1+e^x} \Rightarrow \boxed{D(D+1)y = \frac{1}{1+e^x}}$$

The Auxiliary Equation of above equation is:-

$$D(D+1) = 0.$$

$$\therefore D = 0 \quad D = -1 \qquad \boxed{D = 0, -1}$$

Thus, the $C.F. = y_c = c_1 \cdot e^{0x} + c_2 \cdot e^{-x}$.

$$\boxed{y_c = c_1 + c_2 \cdot e^{-x}} \longrightarrow (1)$$

Here, Particular Integral (P.I.)

$$P.I. = \frac{1}{(D^2+D)} \cdot \frac{1}{(1+e^x)} = \frac{1}{D(D+1)} \cdot \frac{1}{(1+e^x)}$$

$$\therefore P.I. = \left(\frac{1}{D} - \frac{1}{D+1}\right)\left(\frac{1}{1+e^x}\right)$$

$$\therefore P.I. = \frac{1}{D}\left(\frac{1}{1+e^x}\right) - \frac{1}{D+1}\left(\frac{1}{1+e^x}\right)$$

$$\therefore P.I. = \int \frac{1}{1+e^x}\,dx - e^{-x}\int e^x \frac{dx}{1+e^x}$$

$$\therefore P.I. = \int \frac{e^x}{e^x(1+e^x)}\,dx - e^{-x}\int \frac{e^x\,dx}{(1+e^x)}$$

Put, $1 + e^x = t \Rightarrow e^x = t - 1$

$e^x dx = dt$.

$$\therefore P.I. = \int \frac{dx}{t(t-1)} - e^{-x}\int \frac{dt}{t}$$

$$\therefore P.I. = \int\left(\frac{1}{t-1} - \frac{1}{t}\right)dt - e^{-x}\log(t)$$

$$\therefore P.I. = \log(t-1) - \log(t) - e^{-x} \cdot \log(t)$$

$$\therefore P.I. = \log(e^x) - \log(1+e^x) - e^{-x} \cdot \log(1+e^x)$$

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$$P.I. = x - \log(1+e^x) - e^{-x}.\log(e^x+1)$$

$$\boxed{y_p = x - \log(1+e^x) - e^{-x}.\log(e^x+1)} \longrightarrow (2)$$

Hence, The complete solution is:-

$$y = y_c + y_p.$$

$$y = c_1.\cancel{e^{-2x}} + c_2.e^{-x} + x - \log(1+e^x) - e^{-x}.\log(e^x+1)$$

$$\rightarrow \boxed{y = c_1 + c_2.e^{-x} + x - \log(1+e^x) - e^{-x}\log(e^x+1)}$$

Ex 8. SOLVE:- $(D^2 + 3D + 2)y = e^{e^x} + \cos(e^x)$

ANS. SOLUTION:-

The Auxiliary Equation (A.E.) of above given equation is:-

$$D^2 + 3D + 2 = 0.$$

$$(D+2)(D+1) = 0$$

$$\therefore \boxed{D = -2, -1}$$

Thus, $C.F. = y_c = c_1 e^{-2x} + c_2.e^{-x}$

$$\boxed{y_c = c_1.e^{-2x} + c_2.e^{-x}} \longrightarrow (1)$$

Here, particular Intergral (P.I.)

$$P.I. = y_p = \frac{1}{(D+2)(D+1)}(e^{e^x} + \cos e^x)$$

$$\therefore y_p = \frac{1}{(D+2)}\left[e^{-x}\int e^x(e^{e^x} + \cos(e^x)).dx\right]$$

$$\therefore y_p = \frac{1}{(D+2)} e^{-x}(e^{e^x} + \sin e^x)\,dx$$

$$\therefore y_p = e^{-2x}\int e^{2x}.e^{-x}(e^{e^x} + \sin(e^x))\,dx$$

$$\therefore y_p = e^{-2x}\int e^x(e^{e^x} + \sin(e^x))\,dx$$

$$\therefore y_p = e^{-2x}(e^{e^x} - \cos e^x)$$

$$\boxed{y_p = e^{-2x}(e^{e^x} - \cos(e^x))} \longrightarrow (2)$$

$$y = y_c + y_p$$

Hence, The complete solution is!-

$$\rightarrow \boxed{y = c_1.e^{-2x} + c_2.e^{-x} + e^{-2x}(e^{e^x} - \cos.e^x)}$$

Ex 4.) **Solve:-** $(D^2+3D+2)y = \sin e^x$

ANS. SOLUTION:-

The Auxiliary Equation (A.E.) of above given equation is:-

$$D^2 + 3D + 2 = 0.$$

$$(D+2)(D+1) = 0$$

$$\therefore \boxed{D = -2, -1}$$

Thus, $C.F. = y_c = c_1 \cdot e^{-2x} + c_2 \cdot e^{-x}$

$$\boxed{y_c = c_1 \cdot e^{-2x} + c_2 \cdot e^{-x}} \longrightarrow (1)$$

Hence, Particular Integral (P.I.) :-

$$y_p = \frac{1}{(D+2)(D+1)} \cdot \sin e^x = \frac{1}{D+2} e^{-x} \int e^x \cdot \sin e^x \, dx.$$

$$y_p = \frac{1}{(D+2)} \cdot e^{-x}(-\cos e^x) = -e^{-2x} \int e^x \cdot \cos e^x \, dx$$

$$\therefore y_p = -e^{-2x} \cdot \sin(e^x)$$

$$\boxed{y_p = -e^{-2x} \cdot \sin(e^x)} \longrightarrow (2)$$

$$\therefore \quad y = y_c + y_p.$$

Hence, The complete solution is:-

$$y = c_1 \cdot e^{-2x} + c_2 \cdot e^{-x} + (-e^{-2x} \sin(e^x))$$

$$\rightarrow \boxed{y = c_1 \cdot e^{-2x} + c_2 \cdot e^{-x} - e^{-2x} \cdot \sin(e^x)}$$

(B.) Shortcut Method:-

Ex 1.) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = e^{2x}$

ANS. SOLUTION:- $D = d/dx$

$\therefore D^2y - 7Dy + 6y = e^{2x}$

$\therefore (D^2 - 7D + 6)y = e^{2x}$

A.E. : $D^2 - 7D + 6 = 0$

$\boxed{D = 6, 1}$ ....(Roots are Real and Distinct.)

The complimentary Function (C.F.) :-

$y_c = C.F. = c_1 \cdot e^{6 \cdot x} + c_2 \cdot e^{x}$

$\boxed{y_c = c_1 \cdot e^{6x} + c_2 \cdot e^{x}} \longrightarrow (1)$

Particular Integral (P.I.) :-

$y_p = \frac{1}{(D^2 - 7D + 6)} \cdot e^{2x}$

Put, $D = a \quad \therefore D = 2$

$y_p = \frac{1}{(2)^2 - 7(2) + 6} \cdot e^{2x} = \frac{1}{4 - 14 + 6} \cdot e^{2x} = \frac{-1 \cdot e^{2x}}{4}$

$\boxed{y_p = \frac{-e^{2x}}{4}} \longrightarrow (2)$

Hence, the complete solution is:-

$y = y_c + y_p$

A.) $\therefore \boxed{y = c_1 \cdot e^{6x} + c_2 \cdot e^{x} - \frac{e^{2x}}{4}}$

Ex 2.) $\frac{d^2y}{dx^2} - 4y = (1 + e^{x})^2 + 3$

ANS. SOLUTION:- $D = d/dx$

$\therefore D^2y - 4y = (1 + e^{x})^2 + 3$

$\therefore (D^2 - 4)y = (1 + e^{x})^2 + 3$

A.E. : $D^2 - 4 = 0$

$\therefore \boxed{D = 2, -2}$ ....(Roots are Real and Distinct.)

The Complimentary Function (C.F.) :-

$C.F. = y_c = c_1 e^{2x} + c_2 \cdot e^{-2x}$

$\boxed{y = \ldots e^{2x} + \ldots e^{-2x}} \longrightarrow (1)$

Particular Integral: (P.I.) :-

$$y_p = \frac{1}{(D^2-4)} \cdot (1+e^x)^2 + 3 = \frac{1}{(D^2-4)} \; 1 + 2e^x + e^{2x} + 3$$

$$y_p = \frac{1}{(D^2-4)} \left[e^{2x} + 2 \cdot e^x + 4 \right]$$

$$\therefore y_p = \frac{1}{D^2-4} \cdot e^{2x} + \frac{2 \cdot 1}{D^2-4} e^x + \frac{1}{D^2-4} \cdot (4)$$

$$\therefore y_p = \frac{x}{2D-0} \cdot e^{2x} + \frac{2}{(1)^2-4} \cdot e^x + \frac{1}{(0)^2-4} \cdot (4)$$

$$\therefore y_p = \frac{x \cdot e^{2x}}{2(2)} + \frac{2 \cdot e^x}{-3} + \left(\frac{4}{-4}\right)$$

$$\therefore y_p = \frac{x \cdot e^{2x}}{4} - \frac{2 \cdot e^x}{3} - 1$$

$$\boxed{y_p = \frac{x \cdot e^{2x}}{4} - \frac{2 \cdot e^x}{3} - 1} \longrightarrow (2)$$

Hence, The complete solution is:-

$$y = y_c + y_p$$

$$\therefore \boxed{y = c_1 \cdot e^{2x} + c_2 \cdot e^{-2x} + \frac{x \cdot e^{2x}}{4} - \frac{2 \cdot e^x}{3} - 1}$$

Ex 3. $(D^3 - 5D^2 + 8D - 4)y = e^{2x} + 2e^x + 3e^{-x} + 2$

ANS. SOLUTION:-

A.E. : $D^3 - 5D^2 + 8D - 4 = 0$.

$\boxed{D = 1, 2, 2}$ ... (Roots real and repeated)

The complimentary Function (C.F.) :-

$$y_c = \text{C.F.} = c_1 \cdot e^x + (c_2 x + c_3) \cdot e^{2x}$$

$$\boxed{y_c = c_1 \cdot e^x + (c_2 x + c_3) \cdot e^{2x}} \longrightarrow (1)$$

Particular Integral (P.I.):-

$$y_p = \frac{1}{D^3 - 5D^2 + 8D - 4} \cdot (e^{2x} + 2e^x + 3 \cdot e^{-x} + 2)$$

$$\therefore y_p = \frac{1}{D^3 - 5D^2 + 8D - 4} \cdot e^{2x} + \frac{1}{D^3 - 5D^2 + 8D - 4} \cdot 2 \cdot e^x +$$

$$+ \frac{1 \times 3}{D^3 - 5D^2 + 8D - 4} \cdot e^{-x} + \frac{1}{D^3 - 5D^2 + 8D - 4} \cdot (2)$$

$$\therefore y = (y_p)_1 + (y_p)_2 + (y_p)_3 + (y_p)_4$$

$$(y_p)_1 = \frac{x}{3D^2 - 10D + 8} e^{2x} = \frac{x^2 \cdot e^{2x}}{6D - 10} = \frac{x^2 \cdot e^{2x}}{12 - 10}$$

$$\boxed{(y_p)_1 = \frac{x^2 \cdot e^{2x}}{2}} \longrightarrow (i)$$

$$(y_p)_2 = \frac{1 \times 2}{D^3 - 5D^2 + 8D - 4} e^{x} = \frac{2 \cdot x}{3D^2 - 10D + 8} \cdot e^{x} = \frac{2x \cdot e^{x}}{3 - 10 + 8} = \frac{2x \cdot e^{x}}{1}$$

$$\boxed{(y_p)_2 = 2x \cdot e^{x}} \longrightarrow (ii)$$

$$(y_p)_3 = \frac{3}{(-1)^3 - 5(-1)^2 + 8(-1) - 4} \cdot e^{-x} = \frac{\cancel{3}}{-\cancel{18}\,6} \cdot e^{-x} = \frac{-1}{6} \cdot e^{-x}$$

$$\boxed{(y_p)_3 = \frac{-1}{6} \cdot e^{-x}} \longrightarrow (iii)$$

$$(y_p)_4 = \frac{1}{D^3 - 5D^2 + 8D - 4} (2) = \frac{1}{0 - 0 + 0 - 4} (2) = \frac{-2}{4} = \frac{-1}{2}$$

$$\boxed{(y_p)_4 = \frac{-1}{2}} \longrightarrow (iv)$$

$$\boxed{y_p = \frac{x^2}{2} \cdot e^{2x} + 2x \cdot e^{x} - \frac{1}{6} \cdot e^{-x} - \frac{1}{2}} \longrightarrow (2)$$

Hence, The complete solution is:-

$$y = y_c + y_p$$

$$\Rightarrow \boxed{y = c_1 e^{x} + (c_2 \cdot x + c_3) \cdot e^{2x} + \frac{x^2}{2} \cdot e^{2x} + 2x \cdot e^{x} - \frac{1}{6} \cdot e^{-x} - \frac{1}{2}}$$

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Ex4.] $(D^4 - 4D^3 + 6D^2 - 4D + 1)y = e^x + 2^x + \frac{1}{3}$

ANS. SOLUTION:-

A.E. : $D^4 - 4D^3 + 6D^2 - 4D + 1 = 0.$

$\therefore \boxed{D = 1, 1, 1, 1}$

Thus, The roots are real and Repeated.

Complimentary Function:-

$C.F. = y_c = (c_1 \cdot x^3 + c_2 \cdot x^2 + c_3 \cdot x + c_4) \cdot e^x$

$$\boxed{y_c = (c_1 \cdot x^3 + c_2 \cdot x^2 + c_3 \cdot x + c_4) \cdot e^x} \longrightarrow (1.)$$

Particular Integral : (P.I.) :-

$$y_p = \frac{1}{D^4 - 4D^3 + 6D^2 - 4D + 1} (e^x + 2^x + 1/3)$$

$$\therefore y_p = \frac{1}{D^4 - 4D^3 + 6D^2 - 4D + 1} \cdot e^x + \frac{1}{D^4 - 4D^3 + 6D^2 - 4D + 1} \cdot 2^x + \frac{1}{D^4 - 4D^3 + 6D^2 - 4D + 1} \times \frac{1}{3}$$

$$\therefore y_p = (y_p)_1 + (y_p)_2 + (y_p)_3$$

$$(y_{p_1}) = \frac{1}{4D^3 - 12D^2 + 12D - 4} e^x = \frac{1 \; x}{12D^2 - 24D + 12} \cdot e^x$$

$$\therefore (y_p)_1 = \frac{x^2 \; 1}{24D - 24} \cdot e^x = \frac{x^3 \cdot e^x}{24}$$

$$\boxed{(y_p)_1 = \frac{x^3 \cdot e^x}{24}} \longrightarrow (i)$$

$$(y_p)_2 = \frac{1}{D^4 - 4D^3 + 6D^2 - 4D + 1} \cdot 2^x$$

$$(y_p)_2 = \frac{1}{(D-1)(D-1)(D-1)(D-1)} \cdot 2^x = \frac{1}{(D-1)^4} 2^x$$

$$\therefore (y_p)_2 = \frac{1}{(\log 2 - 1)^4} \cdot 2^x$$

$$\boxed{(y_p)_2 = \frac{1}{(\log 2 - 1)^4} \cdot 2^x} \longrightarrow (ii)$$

$$(y_p)_3 = \frac{1}{D^4 - 4D^3 + 6D^2 - 4D + 1} \times \frac{1}{3}$$

Put $D = 0$.

$$\therefore (y_p)_3 = \frac{1}{0 - 0 + 0 - 0 + 1} \times \frac{1}{3} = 1 \times \frac{1}{3} = \frac{1}{3}$$

$$\boxed{(y_p)_3 = \frac{1}{3}} \longrightarrow \text{(iii)}$$

$$y_p = (y_p)_1 + (y_p)_2 + (y_p)_3$$

$$\therefore \boxed{y_p = \frac{x^3}{24} \cdot e^x + \frac{1}{(\log 2 - 1)^4} \cdot 2^x + \frac{1}{3}} \longrightarrow (2)$$

Hence, The complete solution is :-

$$y = y_c + y_p$$

$$\therefore \boxed{y = (c_1 \cdot x^3 + c_2 \cdot x^2 + c_3 \cdot x + c_4) \cdot e^x + \frac{x^3}{24} \cdot e^x + \frac{1}{(\log 2 - 1)^4} \cdot 2^x + \frac{1}{3}}$$

• Extra Questions Math Tut II

a) $(D^2+4)y = (e^x+1)^2$

AE $D^2+4=0$ $\quad D = \pm 2i$

$y_c = C_1 \cos 2x + C_2 \sin 2x$

$$y_p = \frac{1}{D^2+4}(e^x+1)^2$$

$$= \frac{1}{D^2+4}\left(e^{2x} + 1 + 2e^x\right)$$

$$y_{p_1} = \frac{1}{D^2+4}\left(e^{2x}\right) = \frac{1}{4+4}e^{2x} = \frac{e^{2x}}{8}$$

$$y_{p_2} = \frac{1}{4}$$

$$y_{p_3} = 2\left(\frac{1}{D^2+4}\left(e^x\right)\right) = \frac{2}{5}e^x$$

$$y_p = \frac{e^{2x}}{8} + \frac{1}{4} + \frac{2}{5}e^x$$

Soln: $y = y_c + y_p$

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c] Find the proper integral for.

- Trignometric

1.) $\frac{1}{D^2+2D+1}\cos x$

$a = 1$

put $D^2 = -1$

$\therefore \frac{1}{\cancel{-1}+2D\cancel{+1}}\cos x$

$= \frac{1}{2}\int \cos x$

$= \frac{\sin x}{2}$

2.) $\frac{1}{D^3+8}\sin 3x$

→ put $D^2 = -9$

$D^3 = D\cdot D^2 = -9D$

$\therefore = \frac{1}{8-9D}\sin 3x$

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$$\frac{8+9D}{64-81(-9)} \cdot \sin 3x$$

$$= \frac{8+9D}{793} \sin 3x$$

$$y_p = \frac{8\sin 3x}{793} + \frac{3\cos 3x}{81} \quad \text{Ans}$$

• Hyperbolic

1. $\frac{1}{D^2+5} \sinh 2x$

put $D^2 = 4$

$\rightarrow \frac{1}{9} \sinh 2x$

2. $\frac{1}{D^2+2D+1} \cosh x$

$\rightarrow$ put $D^2 = 1$

$= \frac{1}{1+2D+1} \cosh x$

$\frac{1}{2} \frac{1}{D+1} \cosh x$

$= \frac{1}{2} \frac{D-1}{D^2-1} \cosh x$ denominator becomes zero.

$= \frac{1}{2} \cdot x \cdot \frac{(D-1)}{2D} \cosh x$

$= \frac{1}{2} \left[\frac{(D+1)}{-2D} \cosh x \right]$

• Type $e^{ax} \cdot V$

1. $(D^2+2D+1)y = e^{-x} \sin 2x$

$y_p = \frac{1}{D^2+2D+1} e^{-x} \sin 2x$

$a=-1$ $= e^{-x} \frac{1}{(D-1)^2+2(D-1)+1} \sin 2x$

$= e^{-x} \frac{1}{D^2+1-2D+2D-2+1} \sin 2x$

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$$= e^{-x}\frac{1}{D^2}\sin 2x = e^{-x}\left(\frac{-1}{4}\sin 2x\right)$$

Ans

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