Tutorial No. 9

Q1.] Solve following.

1.) Using method of bisection find root of equation $f(x) = x^4 + 2x^3 - x - 1 = 0$ lying in interval [0, 1] at the end of sixth iteration. How many iterations are required if permissible error is $e = 0.0005$

→ Let $f(x) = x^4 + 2x^3 - x - 1$

put $x = 0$

$\therefore f(0) = 0^4 + 2(0)^4 - 0 - 1 = -1 < 0$

put $x = 1$

$\therefore f(1) = 1^4 + 2(1)^4 - 1 - 1 = 1 > 0$

$\therefore$ root lies between 0 & 1

$a = 0, \; b = 1$

$i=1$ $\quad x_1 = \frac{0+1}{2} = \frac{1}{2} = 0.5$

$\therefore f(x_1) = 0.5^4 + 2(0.5)^3 - 0.5 - 1 = -1.1875$

$a = 0.5, \; b = 1$

$i=2$ $\quad x_2 = \frac{0.5+1}{2} = \frac{1.5}{2} = 0.75$

$\therefore f(x_2) = 0.75^4 + 2(0.75)^3 - 0.75 - 1 = -0.5898$

$a = 0.75, \; b = 1$

$i=3$ $\quad x_3 = \frac{0.75+1}{2} = \frac{1.75}{2} = 0.875$

$\therefore f(x_3) = (0.875)^4 + 2(0.875)^3 - 0.875 - 1 = 0.0510$

$a = 0.75, \; b = 0.875$

$i=4 \quad x_4 = \frac{0.75+0.875}{2} = \frac{1.625}{2} = 0.8125$

$f(x_4) = (0.8125)^4 + 2(0.8125)^3 - 0.8125 - 1 = -0.3039$

$a = 0.8125, \; b = 0.875$

$i=5 \quad x_5 = \frac{0.8125+0.875}{2} = 0.84375$

$f(x_5) = (0.84375)^4 + 2(0.84375)^3 - 0.84375 - 1 = -0.1355$

$a = 0.84375, \; b = 0.875$

$i=6 \quad x_6 = \frac{0.84375+0.875}{2} = 0.859375$

$f(x_6) = (0.859375)^4 + 2(0.859375)^3 - 0.859375 - 1 = -0.0446$

number of iterations required be n

$\therefore n \geq \frac{\log(b-a) - \log \epsilon}{\log 2}$

$n \geq \frac{\log(1-0) - \log 0.0005}{\log 2}$

$n \geq 10.9657$

$n = 11$

11 iterations are required if permissible error is 0.0005

2.) find a positive root of equation : $x^3-2x^2+3x-4=0$ at end of fifth iteration by using Secant method.

→ To obtain initial guess, we use intermediate value theorm

$$f(x) = x^3 - 2x^2 + 3x - 4$$

$$f(0) = -4, \quad f(1) = -2, \quad f(2) = 2$$

∴ root lies between 1 & 2, taking initial approximation as $x_0 = 1.00$, $x_1 = 2.0$ we proceed to obtain successive approximations.

putting $i=1$,

$$x_2 = x_1 - \frac{(x_1 - x_0)}{(f_1 - f_0)} \cdot f_1$$

$$f_0 = -2, \quad f_1 = 2$$

$$x_2 = 2 - \frac{(2-1)}{(2+2)} \times 2$$

$$= 1.5$$

$$f(x_2) = (1.5)^3 - 2(1.5)^2 + 3(1.5) - 4 = -0.625$$

$$x_3 = x_2 - \frac{(x_2 - x_1)}{(f_2 - f_1)} \times f_2$$

$$= 1.5 + \frac{(1.5-2)}{(-0.625-2)} \times (-0.625)$$

$$= 1.5 + \frac{0.5}{2.625} \times 0.625$$

$$= 1.619$$

$$f(x_3) = (1.619)^3 - 2(1.619)^2 + 3(1.619) - 4$$

$$= -0.1417$$

$$x_4 = x_3 - \frac{(x_3 - x_2)}{(f_3 - f_2)} \times f_3$$

$$= 1.619 - \frac{(1.619 - 1.5)}{(-0.1417 + 0.625)} \times (-0.1417)$$

$$= 1.619 + \frac{0.119}{0.4833} \times 0.1417$$

$$= 1.6539$$

$$= 1.654$$

$$f(x_4) = (1.654)^3 - 2(1.654)^2 + 3(1.654) - 4 = 0.015$$

$$x_5 = x_4 - \frac{(x_4 - x_3)}{(f_4 - f_3)} \times f_4$$

$$= 1.654 - \frac{(1.654 - 1.619)}{(0.015 + 0.1417)} \times 0.015$$

$$= 1.654 - \frac{0.035}{0.1567} \times 0.015$$

$$= 1.6506$$

$$f(x_5) = (1.6506)^3 - 2(1.6506)^2 + 3(1.6506) - 4 = -0.00013$$

$$x_6 = x_5 - \frac{(x_5 - x_4)}{(f_5 - f_4)} \times f_5$$

$$= 1.6506 - \frac{(1.6506 - 1.6540)}{(-0.00013 - 0.0015)} \times (-0.00013)$$

$$= 1.6506 + \frac{0.0034}{0.01513} \times 0.00013 = 1.65063$$

$$= 1.65063$$

correct to four decimal places.

3) find real root of equation $x^3-2x-5=0$ by method of false position at end of sixth iteration.

$\Rightarrow$ $f(x) = x^3-2x-5=0$

$f(0) = -5$

$f(1) = -6$

$f(2) = -1$

$f(3) = 16$

$\therefore$ root lies between 2 & 3.

Taking, $x_0 = 2$, $f_0 = -1$ & $x_1 = 3$, $f_1 = 16$

$$x_2 = x_1 - \frac{(x_1 - x_0)}{(f_1 - f_0)} \times f_1$$

$$= 3 - \frac{(3-2)}{(16-(-1))} \times 16$$

$$= 3 - \frac{1}{17} \times 16$$

$$= 2.0588$$

$f(x_2) = (2.0588)^3 - 2(2.0588) - 5 = -0.3908$

Now we take interval $(2.0588, 3)$ as $f_2 \times f_1 < 0$

Here, $x_2 = 2.0588$, $f_2 = -0.3908$

let $x_1 = 3$, $f_1 = 16$

$$x_3 = x_2 - \frac{(x_2 - x_1)}{(f_2 - f_1)} \times f_2$$

$$= 2.0588 - \frac{(2.0588 - 3)}{(-0.3908 - 16)} \times (-0.3908)$$

$$= 2.0588 + 0.0224$$

$$= 2.0812$$

$f(x_3) = -0.1479$

interval (2.0812, 3) as $f_3 \times f_1 < 0$

Here $x_3 = 2.0812$ $f_3 = -0.1479$

$x_2 = 3$ so that $f_2 = 16$

$$x_4 = x_3 - \frac{(x_3 - x_2)}{(f_3 - f_2)} f_3$$

$$= 2.0812 - \frac{(2.0812 - 3)}{(-0.1479 - 16)} \times (-0.1479)$$

$$= 2.0812 + 0.00842$$

$$= 2.0896$$

$f(x_4) = -0.0511$

interval (2.0896, 3) as $f_4 \times f_1 < 0$

$x_4 = 2.0896$ $f_4 = -0.0577$

$x_3 = 3$ $f_3 = 16$

$$x_5 = x_4 - \frac{(x_4 - x_3)}{(f_4 - f_3)} f_4$$

$$= 2.0896 - \frac{(2.0896 - 3)}{(-0.0511 - 16)} \times (-0.0511)$$

$$= 2.0896 + 0.0029$$

$$= 2.0925$$

$f(x_5) = -0.0229$

interval (2.0925, 3) as $f_5 \times f_1 < 0$

$x_5 = 2.0925$; $f_5 = -0.0229$

$x_4 = 3$, $f_4 = 16$

$$x_6 = 2.0925 - \frac{(2.0925 - 3)}{(-0.0229 - 16)} \times (-0.0229)$$

$$= 2.0925 + 0.0013$$

$$= 2.0938$$

Q4.) find the root of equation $x^3-4x-9=0$, correct to four decimal places by using Newton-Raphson method

→

$f(x) = x^3-4x-9$

$f(0) = -9$

$f(1) = -12$

$f(2) = -9$

$f(3) = 6$

∴ root lies between 2 & 3.

To choose initial value consider,

$f'(x) = 3x^2-4, \quad f''(x) = 6x, \quad f''(x) = 18$

since $f(3) = 6$ and $f''(3) = 18$ are the same sign we choose initial approximation as $x_0 = 3$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{f(3)}{f''(3)} = 3 - \frac{6}{23} = 2.739$$

$f(x_1) = f(2.739) = (2.739)^3 - 4(2.739) - 9 = 0.59231$

$f'(x_1) = f'(2.739) = 3(2.739)^2 - 4 = 18.5064$

$$x_2 = x_1 - \frac{x_1}{f'(x_1)} = 2.739 - \frac{0.59231}{18.5064} = 2.707$$

$f(x_2) = f(2.707) = (2.707)^3 - 4(2.707) - 9 = 0.008487$

$f'(x_2) = f'(2.707) = 3(2.707)^2 - 4 = 17.983547$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.707 - \frac{0.008487}{17.983547} = 2.706528$$

$f(x_3) = (2.706528)^3 - 4(2.706528) - 9 = 8.179 \times 10^{-7}$

root = 2.7065

5.) Solve $3x-1-\cos x = 0$ by using method of successive approximation correct to three decimal places.

→ let $f(x) = 3x-1-\cos x$

$f(0) = -2$

$f(1) = 1.459696$

∴ Root lies between 0 & 1 (Radians)

$x = \phi(x)$

$x = \frac{1+\cos x}{3} = \phi(x)$

$\phi'(x) = \frac{1}{3}\sin x$ & $f'(x) = \left|\frac{-\sin x}{3}\right| < 1$ in $(0,1)$

Method of approximation will give convergent result.

$x_0 = 0$

$x_1 = \frac{1+\cos 0}{3} = \frac{2}{3} = 0.6667$

$x_2 = \frac{1+\cos(0.6667)}{3} = 0.5953$

$x_3 = \frac{1+\cos(0.5953)}{3} = 0.6093$

$x_4 = \frac{1+\cos(0.6093)}{3} = 0.6067$

$x_5 = \frac{1+\cos(0.6067)}{3} = 0.6072$

$x_6 = \frac{1+\cos(0.6072)}{3} = 0.6071$

x_5 & x_6 are same.