End Semester Examinations May-June 2017

MES College of Engineering
SE COMP

Engineering Mathematics III
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1. (a) Solve any two of the following: [8]

i. $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ by using Variation of parameters

Solution:

$$D^2 - 1 = 0, \therefore D = \pm 1$$
$$y_c = c_1 e^x + c_2 e^{-x}$$
$$\therefore y_p = ue^x + ve^{-x}$$

Now $W = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -2$

$$u = \int \frac{-y_2 f(x)}{W} dx$$
$$= \int \frac{-e^{-x}\frac{2}{1+e^x}}{-2} dx$$
$$= \int \frac{e^{-2x}}{e^{-x}+1} dx$$

Substituting $1 + e^{-x} = t$ gives

$$u = 1 + e^{-x} - \log(1 + e^{-x})$$

Now

$$v = \int \frac{y_1 f(x)}{W} dx$$
$$= \int \frac{e^x \frac{2}{1+e^x}}{-2} dx$$
$$= -\log(1 + e^x)$$

$$y_p = 1 + e^{-x} - \log(1 + e^{-x})e^x + -\log(1 + e^x)e^{-x}$$

Hence the complete solution is given by $y = y_c + y_p$

ii. $(D^2-4)y = e^{2x}+2x^3$

Solution: Auxillary equation is $D^2-4=0 \therefore D=\pm 2$

$$\therefore y_c = c_1e^{2x}+c_2e^{-2x}$$

$$y_p = \frac{1}{D^2-4}(e^{2x}+2x^3) = y_{p_1}+y_{p_2}$$

Let us find

$$\begin{aligned} y_{p_1} &= \frac{1}{D^2-4}e^{2x} \\ &= \frac{x}{2D}e^{2x} \ldots \text{Putting D=2 we get denominator =0} \\ &= \frac{x}{4}e^{2x} \ldots \text{Put D=2} \end{aligned}$$

and

$$\begin{aligned} y_{p_2} &= \frac{1}{D^2-4}2x^3 \\ &= \frac{1}{-4(1-\frac{D^2}{4})}2x^3 \ldots \text{Use Series expansion} \\ &= \frac{-1}{2}(1+\frac{D^2}{4})x^3 \\ &= \frac{-1}{2}(x^3+\frac{3}{2}x) \end{aligned}$$

$$\boxed{\therefore y_p = \frac{x}{4}e^{2x}+\frac{-1}{2}(x^3+\frac{3}{2}x)}$$

Hence the complete solution is given by $y=y_c+y_p$

iii. $(2x+1)^2\frac{d^2y}{dx^2}-2(2x+1)\frac{dy}{dx}-12y=24x$

Solution: Put $2x+1=e^z \therefore x=\frac{e^z-1}{2}$
$(2x+1)^2\frac{d^2y}{dx^2}=4D(D-1)y, (2x+1)\frac{dy}{dx}=2Dy$ where $D=\frac{d}{dz}$
Hence the new equation becomes

$(D^2 - 2D - 3)y = 3(e^z - 1)$

$$
\begin{aligned}
D^2 - 2D - 3 &= 0 \\
\therefore D &= -3, 1 \\
y_c &= c_1 e^{-3z} + c_2 e^z \\
y_p &= \frac{1}{D^2 - 2D - 3} 3(e^z - 1) \\
&= \frac{1}{D^2 - 2D - 3} 3e^z + \frac{3}{D^2 - 2D - 3} \\
&= \frac{-3}{4} e^z - 1
\end{aligned}
$$

2. Solve the following integral equation using Fourier Transform [4]

$\int_0^\infty f(x) \sin \lambda x d\lambda = \begin{cases} 1 - \lambda, \ 0 \leq \lambda \leq 1 \\ 0, \qquad \lambda \geq 1 \end{cases}$

Solution: By using Inverse Fourier Sine Transform

$$
\begin{aligned}
f(x) &= \frac{2}{\pi} \int_0^\infty F_s(\lambda) \sin \lambda x d\lambda \\
&= \frac{2}{\pi} \left(\int_0^1 (1 - \lambda) \sin \lambda x d\lambda + \int_1^\infty 0 \sin \lambda x d\lambda \right) \\
&= \frac{2}{\pi} \left[(1 - \lambda) \frac{-\cos \lambda x}{x} - \frac{\sin \lambda x}{x^2} \right]_0^1 \\
&= \frac{2}{\pi} \left(\frac{\sin x}{x^2} + \frac{1}{x} \right)
\end{aligned}
$$

1. (a) Solve any two of the following: [8]

i. $(D^2+4)y = e^x + x^2$

Solution: Auxillary equation is $D^2+4=0 \therefore D = \pm 2i$

$$\therefore y_c = c_1 \cos 2x + c_2 \sin 2x$$

$$y_p = \frac{1}{D^2+4}(e^x + x^2) = y_{p_1} + y_{p_2}$$

$$\begin{aligned} y_{p_1} &= \frac{1}{D^2+4} e^x \\ &= \frac{1}{5} e^x \ldots Put D = 1 \\ y_{p_2} &= \frac{1}{D^2+4} x^2 \\ &= \frac{1}{4(1+\frac{D^2}{4})} x^2 \\ &= \frac{1}{4}(1 - \frac{D^2}{4}) x^2 \\ &= \frac{1}{4}(x^2 - \frac{1}{2}) \end{aligned}$$

$$\boxed{y_p = \frac{1}{5}e^x + \frac{1}{4}(x^2 - \frac{1}{2})}$$

Hence the complete solution is given by $y = y_c + y_p$.

ii. $(D^2+6D+9)y = x^{-4}e^{-3x}$

Solution: Auxillary equation is $D^2+6D+9=0 \therefore D = -3,-3$

$$\therefore y_c = (c_1+c_2x)e^{-3x}$$

$$\begin{aligned} y_p &= \frac{1}{D^2+6D+9}(x^{-4}e^{-3x}) \\ &= e^{-3x}\frac{1}{(D-3)^2+6(D-3)+9}x^{-4} \ldots \text{Replace D by D-3} \\ &= e^{-3x}\frac{1}{D^2}x^{-4} \\ &= e^{-3x}\int\int x^{-4}dxdx \\ &= e^{-3x}\frac{1}{6x^2} \end{aligned}$$

Hence the complete solution is given by $y = y_c + y_p$

iii. $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{x}\sin(\log x)$

Solution: Put $x = e^z \therefore z = \log x$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y, x\frac{dy}{dx} = Dy$$

where $D = \frac{d}{dz}$
Hence the new equation becomes
$(D^2+2D+1)y = e^{-z}\sin z$

$$
\begin{aligned}
D^2 + 2D + 1 &= 0 \\
\therefore D &= -1, -1 \\
y_c &= (c_1 + c_2 z)e^{-z} = (c_1 + c_2 \log x)\frac{1}{x} \\
y_p &= \frac{1}{D^2 + 2D + 1} e^{-z} \sin z \\
&= e^{-z} \frac{1}{(D-1)^2 + 2(D-1) + 1} \sin z \\
&= e^{-z} \frac{1}{D^2} \sin z \\
&= e^{-z} \frac{1}{-1} \sin z = -\frac{1}{x} \sin(\log x)
\end{aligned}
$$

Hence the complete solution is given by $y = y_c + y_p$

2. Solve the following integral equation using Fourier Transform [4]

$$\int_0^\infty f(x) \sin \lambda x d\lambda = \begin{cases} 1 - \lambda, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1 \end{cases}$$

Solution: By using Inverse Fourier Transform

$$
\begin{aligned}
f(x) &= \frac{2}{\pi} \int_0^\infty F_s(\lambda) \sin \lambda x d\lambda \\
&= \frac{2}{\pi} \left(\int_0^1 (1 - \lambda) \sin \lambda x d\lambda + \int_1^\infty 0 \sin \lambda x d\lambda \right) \\
&= \frac{2}{\pi} \left[(1 - \lambda) \frac{-\cos \lambda x}{x} - \frac{\sin \lambda x}{x^2} \right]_0^1 \\
&= \frac{2}{\pi} \left(\frac{\sin x}{x^2} + \frac{1}{x} \right)
\end{aligned}
$$

1. (a) Solve any two of the following: [8]

i. $(D^2 - 4D + 4)y = e^{2x} \sin 4x + 2^{3x} + 6$

Solution: Auxillary equation is $D^2 - 4D + 4 = 0 \therefore D = 2, 2$

$$\therefore y_c = (c_1 + c_2 x)e^{2x}$$

$$\begin{aligned} y_p &= \frac{1}{D^2 - 4D + 4}(e^{2x} \sin 4x + 2^{3x} + 6) \\ &= y_{p_1} + y_{p_2} + y_{p_3} \end{aligned}$$

Now

$$\begin{aligned} y_{p_1} &= \frac{1}{D^2 - 4D + 4} e^{2x} \sin 4x \\ &= e^{2x} \frac{1}{(D+2)^2 - 4(D+2) + 4} \sin 4x \\ &= e^{2x} \frac{1}{D^2} \sin 4x \\ &= e^{2x} \frac{1}{-16} \sin 4x \end{aligned}$$

$$\begin{aligned} y_{p_2} &= \frac{1}{D^2 - 4D + 4} 2^{3x} \\ &= \frac{1}{(\log 8)^2 - 4\log 8 + 4} 2^{3x} \ldots Put\, D = \log 8 \end{aligned}$$

$$\begin{aligned} y_{p_3} &= \frac{1}{D^2 - 4D + 4} 6 \\ &= \frac{1}{4} 6 \\ &= \frac{3}{2} \ldots Put\, D = 0 \end{aligned}$$

$$\boxed{\therefore y_p = e^{2x}\frac{1}{-16}\sin 4x + \frac{1}{(\log 8)^2 - 4\log 8 + 4}2^{3x} + \frac{3}{2}}$$

Hence the complete solution is given by $y = y_c + y_p$

ii. $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^4 + 3x + 1$

Solution: Solution. Put $x = e^z \therefore z = \log x$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y, x\frac{dy}{dx} = Dy$$

where $D = \frac{d}{dz}$
Hence the new equation becomes

$$(D^2 + 1)y = e^{4z} + 3e^z + 1$$

Auxillary equation is $D^2 + 1 = 0 \therefore D = \pm i$

$$\therefore y_c = c_1\cos z + c_2\sin z = c_1\cos(\log x) + c_2\sin(\log x)$$

$$\begin{aligned} y_{p_1} &= \frac{1}{D^2+1}e^{4z} + 3e^z + 1 \\ &= \frac{1}{D^2+1}e^{4z} + \frac{1}{D^2+1}3e^z + \frac{1}{D^2+1}1 \\ &= \frac{1}{17}e^{4z} + \frac{1}{2}3e^z + 1 \\ &= \frac{1}{17}x^4 + \frac{1}{2}3x + 1 \end{aligned}$$

Hence the complete solution is given by $y = y_c + y_p$

iii. $\frac{d^2y}{dx^2} + 9y = \tan 3x$ by using Variation of parameters

Solution: Solution. Characteristic Equation is $D^2 + 9 = 0$
$\therefore D = \pm 3i$
$\therefore y_c = c_1\cos 3x + c_2\sin 3x$

$\therefore y_p = u\cos 3x + v\sin 3x$

Now $W = \begin{vmatrix} \cos 3x & \sin 3x \\ 3sin3x & 3\cos 3x \end{vmatrix} = 3$

$$u = \int \frac{-y_2 f(x)}{W} dx = -\int \frac{\sin 3x \tan 3x}{3} dx = -\frac{1}{3}\int \frac{\sin^2 3x}{\cos 3x} dx$$

$$u = -\frac{1}{3}\int \frac{1-\cos^2 3x}{\cos 3x} dx = -\frac{1}{3}\int (\sec 3x - \cos 3x) dx$$

$$u = -\frac{1}{9}(\log(\sec 3x + \tan 3x) - \sin 3x)$$

$$v = \int \frac{-y_1 f(x)}{W} dx = \int \frac{\cos 3x \tan 3x}{3} dx = \frac{1}{3}\int \sin 3x dx$$

$$v = \frac{-1}{9}\cos 3x$$

$\therefore y_p = -\frac{1}{9}(\log(\sec 3x + \tan 3x) - \sin 3x)\cos 3x + \frac{-1}{9}\cos 3x \sin 3x$

Hence the complete solution is given by $y = y_c + y_p$

2. Solve the following integral equation using Fourier Transform [4]

$\int_0^\infty f(x)\cos \lambda x d\lambda = \begin{cases} 2-\lambda, & 0 \le \lambda \le 2 \\ 0, & \lambda \ge 2 \end{cases}$

Solution: By using Inverse Fourier Cosine Transform

$$\begin{aligned} f(x) &= \frac{2}{\pi}\int_0^\infty F_s(\lambda)\cos\lambda x d\lambda \\ &= \frac{2}{\pi}\left(\int_0^2 (2-\lambda)\cos\lambda x d\lambda + \int_2^\infty 0\cos\lambda x d\lambda\right) \\ &= \frac{2}{\pi}\left[(2-\lambda)\frac{\sin\lambda x}{x} + \frac{\cos\lambda x}{x^2}\right]_0^2 \\ &= \frac{2}{\pi}\left(\frac{\cos 2x}{x^2} - \frac{1}{x^2}\right) \end{aligned}$$

End Semester Examinations Nov-Dec 2017

MES College of Engineering
SE COMP

Engineering Mathematics III

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1. (a) Solve any two of the following: [8]

   i. $(D^2 + 3D + 2)y = 2e^{e^x}$

   Solution: Auxillary equation is $D^2 + 3D + 2 = 0 \therefore D = -1, -2$

   $$\therefore y_c = c_1 e^{-x} + e^{-2x}$$

$$
\begin{aligned}
y_p &= \frac{1}{D^2+3D+2} 2e^{e^x} \\
&= \frac{1}{(D+1)(D+2)} 2e^{e^x} \\
&= 2\frac{1}{(D+2)} \left(\frac{1}{(D+1)} e^{e^x} \right) \\
&= 2\frac{1}{(D+2)} \left(e^{-x} \int e^x e^{e^x} dx \right) \\
&= \frac{1}{(D+2)} e^{-x} e^{e^x} \\
&= 2e^{-2x} \int e^{2x} e^{-x} e^{e^x} dx \\
&= 2e^{-2x} \int e^x e^{e^x} dx
\end{aligned}
$$

$$y_p = 2e^{-2x} e^{e^x}$$

Hence the complete solution is given by $y = y_c + y_p$

ii. $(D^2 + 4D + 4)y = x^{-3}e^{-2x}$

Solution: Auxillary equation is $D^2 + 4D + 4 = 0 \therefore D = -2, -2$

$$\therefore y_c = (c_1 + c_2 x)e^{-2x}$$

$$
\begin{aligned}
y_p &= \frac{1}{D^2+4D+4}(x^{-3}e^{-2x}) \\
&= e^{-2x}\frac{1}{(D-2)^2+4(D-2)+4}x^{-3} \\
&= e^{-2x}\frac{1}{D^2}x^{-3} \\
&= e^{-2x}\int\int x^{-3}dxdx \\
&= e^{-2x}\frac{1}{2x}
\end{aligned}
$$

Hence the complete solution is given by $y = y_c + y_p$

iii. $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2\sin(\log x)$

Solution: Put $x = e^z \therefore z = \log x$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y, x\frac{dy}{dx} = Dy$$

where $D = \frac{d}{dz}$
Hence the new equation becomes
$(D^2 - 4D + 5)y = e^{2z}\sin z$

$$
\begin{aligned}
&D^2 - 4D + 5 = 0\\
&\qquad \therefore D = 2 \pm i\\
&\qquad y_c = e^{2z}(c_1 \cos z + c_2 \sin z) = x^2(c_1 \cos(\log x) + c_2 \sin \log x)\\
&\qquad y_p = \frac{1}{D^2 - 4D + 5} e^{2z} \sin z\\
&\qquad\quad = e^{2z} \frac{1}{(D+2)^2 - 4(D+2) + 5} \sin z\\
&\qquad\quad = e^{2z} \frac{1}{D^2 + 1} \sin z \ldots Put D^2 = -1\\
&\qquad\quad = e^{2z} \frac{z}{2D} \sin z \ldots \text{since Denominator=0}\\
&\qquad\quad = e^{2z} \frac{z}{2} \int \sin z dz\\
&\qquad\quad = -e^{2z} \frac{z}{2} \cos z\\
&\qquad\quad = \frac{-x^2 \log x \cos(\log x)}{2}
\end{aligned}
$$

Hence the complete solution is given by $y = y_c + y_p$

2. Find the Fourier Transform of [4]

$$f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \quad \text{and evaluate } \int_0^\infty \frac{\sin \lambda}{\lambda} d\lambda$$

Solution: Since the given function is an even function

By Fourier Cosine Transform

$$\begin{aligned} F_c(\lambda) &= \int_0^\infty f(x)\cos\lambda x dx \\ &= \int_0^1 1\cos\lambda x dx + \int_1^\infty 0\cos\lambda x dx \\ &= \left[\frac{\sin\lambda x}{\lambda}\right]_0^1 \\ &= \frac{\sin\lambda}{\lambda} \end{aligned}$$

By Using Inverse Fourier Cosine Transform

$$f(x) = \frac{2}{\pi}\int_0^\infty F_c(\lambda)\cos\lambda x d\lambda$$

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{\sin\lambda}{\lambda}\cos\lambda x d\lambda$$

Put $x = 0$ we get

$$\int_0^\infty \frac{\sin\lambda}{\lambda}d\lambda = \frac{\pi}{2}$$

End Semester Examinations Nov-Dec 2018

MES College of Engineering
SE COMP

Engineering Mathematics III

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1. (a) Solve any two of the following: [8]

   i. $(D^2 + 2D + 1)y = e^{-x}x\cos x$

Solution: Auxillary equation is $D^2 + 2D + 1 = 0 \therefore D = -1, -1$

$$\therefore y_c = (c_1 + xc_2)e^{-x}$$

$$\begin{aligned}
y_p &= \frac{1}{D^2+2D+1} e^{-x} x \cos x \\
&= e^{-x} \frac{1}{(D-1)^2 + 2(D-1) + 1} x \cos x \\
&= e^{-x} \frac{1}{D^2} x \cos x \\
&= e^{-x} \left[x - \frac{2D}{D^2} \right] \frac{1}{D^2} \cos x \\
&= e^{-x} \left[x - \frac{2D}{-1} \right] \frac{1}{-1} \cos x \ldots Put D^2 = -1 \\
&= -e^{-x} \left[x + 2D \right] \cos x \\
&= -e^{-x} \left[x \cos x - 2 \sin x \right]
\end{aligned}$$

Hence the complete solution is given by $y = y_c + y_p$

ii. $(2x+1)^2 \frac{d^2y}{dx^2} + 2(2x+1)\frac{dy}{dx} + 4y = 4\sin[2\log(2x+1)]$

Solution: Put $2x + 1 = e^z \therefore z = \log(2x+1)$
$(2x+1)^2 \frac{d^2y}{dx^2} = 4D(D-1)y, (2x+1)\frac{dy}{dx} = 2Dy$ where $D = \frac{d}{dz}$
Hence the new equation becomes
$(D^2 + 1)y = \sin 2z$

$$D^2 + 1 = 0$$

$$\therefore D = \pm i$$

$$y_c = c_1 \cos z + c_2 \sin z = c_1 \cos(\log(2x+1)) + c_2 \sin(\log(2x+1))$$

$$\begin{aligned} y_p &= \frac{1}{D^2+1} \sin 2z \\ &= \frac{1}{-4+1} \sin 2z \ldots Put D^2 = -4 \\ &= -\frac{1}{3} \sin 2z = -\frac{1}{3} \sin 2(\log(2x+1)) \end{aligned}$$

Hence the complete solution is given by $y = y_c + y_p$

iii. $(D^2 + 3D + 2)y = 2e^{e^x}$ by using variation of parameters

Solution: Auxillary equation is $D^2 + 3D + 2 = 0$

$$\therefore D = -1, -2$$

$$\therefore y_c = c_1 e^{-x} + e^{-2x}$$

Now $W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -3e^{-3x}$

$$\begin{aligned} u &= \int \frac{-y_2 f(x)}{W} dx \\ &= -\int \frac{e^{-2x} 2e^{e^x}}{-3e^{-3x}} dx \\ &= -\frac{2}{3} \int e^x e^{e^x} dx = -\frac{2}{3} e^{e^x} \end{aligned}$$

$$\begin{aligned} v &= \int \frac{y_1 f(x)}{W} dx \\ &= -\int \frac{e^x 2e^{e^x}}{-3e^{-3x}} dx \\ &= -\frac{2}{3} \int e^{2x} e^{e^x} dx = -\frac{2}{3}(e^x - 1)e^{e^x} \end{aligned}$$

Hence the complete solution is given by $y = y_c + y_p$

2. Solve the integral equation [4]

$\int_0^{\infty} f(x) \sin \lambda x dx = e^{-\lambda}, \lambda > 0$

Solution: Since the given function is an even function
By Fourier Cosine Transform

$$\begin{aligned} f(x) &= \int_0^{\infty} F_s(\lambda) \sin \lambda x \, d\lambda \\ &= \int_0^{\infty} e^{-\lambda} \sin \lambda x \, d\lambda \\ &= \left[\frac{e^{-\lambda}}{x^2+1} (-\sin \lambda x - x \cos \lambda x) \right]_0^{\infty} \\ &= \frac{x}{x^2+1} \end{aligned}$$

End Semester Examinations Nov-Dec 2019

MES College of Engineering
SE COMP

Engineering Mathematics III

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1. (a) Solve any two of the following: [8]

i. $(D^2 + 7D - 2)y = e^{4x} \cosh 2x$

Solution: Auxillary equation is $D^2 + 7D - 2 = 0 \therefore D = \frac{-7 \pm \sqrt{57}}{2} = 0.27, -11.04$

$$\therefore y_c = c_1 e^{0.27x} + c_2 e^{-11.04x}$$

$$\begin{aligned} y_p &= \frac{1}{D^2+7D-2} e^{4x} \cosh 2x \\ &= e^{4x} \frac{1}{(D+4)^2 + 7(D+4) - 2} \cos h2x \\ &= e^{4x} \frac{1}{D^2 + 15D + 42} \cosh 2x \\ &= e^{4x} \frac{1}{4 + 15D + 42} \cosh 2x \ldots Put D^2 = 4 \\ &= e^{4x} \frac{1}{15D + 46} \cosh 2x \\ &= e^{4x} \frac{15D - 46}{225D^2 - 2116} \cosh 2x \\ &= e^{4x} \frac{15D - 46}{-1216} \cosh 2x \\ &= e^{4x} \left(\frac{30 \sinh 2x - 46 \cosh 2x}{-1216} \right) \end{aligned}$$

Hence the complete solution is given by $y = y_c + y_p$

ii. $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^2 \sin(\log x)$

Solution: Put $x = e^z \therefore z = \log x$

$$x^2 \frac{d^2y}{dx^2} = D(D-1)y, x\frac{dy}{dx} = Dy$$

where $D = \frac{d}{dz}$
Hence the new equation becomes
$(D^2 - 4D + 3)y = e^{2z} \sin z$

$$D^2 - 4D + 3 = 0$$

$$\therefore D = -3, -1$$

$$y_c = (c_1 e^{-3z} + c_2 e^{-z}) = (c_1 x^{-3} + c_2 x^{-1})$$

$$y_p = \frac{1}{D^2 - 4D + 3} e^{2z} \sin z$$

$$= e^{2z} \frac{1}{(D+2)^2 - 4(D+2) + 3} \sin z$$

$$= e^{2z} \frac{1}{D^2 - 1} \sin z \ldots Put D^2 = -1$$

$$= e^{2z} \frac{1}{-2} \sin z$$

$$= -\frac{x^2 \sin(\log x)}{2}$$

Hence the complete solution is given by $y = y_c + y_p$

iii. $(D^2 - 8D + 16)y = \frac{e^{4x}}{x^6}$ by using variation of parameters

Solution: Auxillary equation is $D^2 - 8D + 16$

$$\therefore D = 4, 4$$

$$\therefore y_c = (c_1 + +c_2 x)e^{4x}$$

Now $W = \begin{vmatrix} e^{4x} & xe^{4x} \\ 4e^{4x} & e^{4x}(4x+1) \end{vmatrix} = e^{8x}$

$$u = \int \frac{-y_2 f(x)}{W} dx$$

$$= -\int \frac{xe^{4x} e^{4x}}{e^{8x} x^6} dx$$

$$= -\int \frac{1}{x^5} dx = \frac{x^{-4}}{4}$$

$$
\begin{aligned}
v &= \int \frac{y_1 f(x)}{W} dx \\
&= \int \frac{e^{4x} e^{4x}}{e^{8x} x^6} dx \\
&= \int \frac{1}{x^6} dx \\
&= \frac{x^{-5}}{-5}
\end{aligned}
$$

Hence the complete solution is given by $y = y_c + y_p$