Modern Education Society's

Wadia College of Engineering, Pune 411 001

Tutorial / Test Examination

Day and Date 20/4/2024. Subject Regression Analysis.

Class SE COMP Sem.

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Roll No.

Signature of Supervisor

Marks Obtained Signature of Examiner

1. The equations of the regression lines between two variables are expressed as $2x-3y=0$ and $4y-5x-8=0$. Find $\bar{x}$ and $\bar{y}$, the regression coefficients and the correlation coefficient between x and y.

sol^n:-

The two regression equations are

$$2x-3y=0 \quad \text{———} \ (1)$$

$$4y-5x-8=0 \quad \text{———} \ (2)$$

The point $(\bar{x}, \bar{y})$ lies at the intersection of both the lines of regression equations. Solving the two regression equations.

$\therefore eq^n (1) \times 4 \Rightarrow 8\bar{x} - 12\bar{y} = 0$

$eq^n (2) \times 3 \Rightarrow -15\bar{x} + 12\bar{y} - 24 = 0$

$$\therefore \bar{x} = -3.429, \quad \bar{y} = -2.286.$$

From eq^n (1), $y = \frac{2}{3}x \Rightarrow b_{yx} = \frac{2}{3}$

From eq^n (2), $x = \frac{4}{5}y - \frac{8}{5} \Rightarrow b_{xy} = \frac{4}{5}$

$\therefore r^2 = b_{yx} \times b_{xy} = \frac{8}{15} \Rightarrow r = 0.73$

(r is positive since b_{yx} and b_{xy} are positive)

Q.2 The two regression lines obtained from certain data were : $y = x+5$ and $16x = 9y - 94$. Find the variance of x, if the variance of y is 16. Also find the covariance between x and y.

Soln:- The two regression lines are given by

$$y = x+5$$
$$16x = 9y - 94$$

$$\Rightarrow x = \frac{9}{16}y - \frac{47}{8}$$

$$\therefore b_{yx} = 1 \quad , \quad b_{xy} = \frac{9}{16}$$

Also, $r^2 = b_{yx} \times b_{xy} \Rightarrow r = \frac{3}{4}$

$$\therefore \quad b_{xy} = r\frac{\sigma_x}{\sigma_y} \Rightarrow 16 = \frac{6}{4}\frac{\sigma_x}{\sqrt{16}}$$

$$\Rightarrow \quad \sigma_x = 3$$

$$\therefore \quad r = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$\therefore \quad cov(x,y) = r\sigma_x\sigma_y = \frac{3}{4} \times 3 \times 4 = 9$$

Q.3. Find the mean value of random variable x and y and the correlation coefficient between them, when the two lines of regression are given by :

$$5x + 7y - 22 = 0 \quad \text{and} \quad 6x + 2y - 20 = 0$$

If the variance of y is 15, find the standard deviation of x.

Sol^n:- Point $(\bar{x}, \bar{y})$ satisfies the given equations

$$\therefore \quad 5\bar{x}+7\bar{y}=22$$
$$6\bar{x}+2\bar{y}=20$$

Solving these equations $\bar{x}=3, \bar{y}=1$.

$$5x+7y=22 \Rightarrow y=\frac{22}{7}-\frac{5}{7}x$$

$$\therefore b_{yx}=-\frac{5}{7}$$

$$6x+2y=20 \Rightarrow x=-\frac{1}{3}y+\frac{10}{3}$$

$$\therefore b_{xy}=-\frac{2}{6}$$

$$\therefore r=\sqrt{b_{yx} \cdot b_{xy}}=-\sqrt{\frac{10}{42}}=-\sqrt{\frac{5}{21}}$$

[$\because$ b_{yx}, b_{xy} are negative, r is negative]

Again, $\sigma_y=\sqrt{15}$

$$b_{xy}=r\frac{\sigma_x}{\sigma_y} \Rightarrow -\frac{1}{3}=\left(-\sqrt{\frac{5}{21}}\right)\frac{\sigma_x}{\sigma_y}$$

$$\Rightarrow \sigma_x=\frac{1}{3}\sqrt{\frac{21}{5}}\times\sqrt{15}=2.646.$$

Hence standard deviation of x is 2.646.

Q.4. Given that the variance of $x=9$ and the regression equations are

$$8x-10y+66=0$$
$$40x-18y=214$$

Find (a) mean values of x and y

(b) coefficient of correlation between x & y.

(c) Standard deviation of y.

Soln:- (a) The mean values of x and y ie $(\bar{x}, \bar{y})$ lie on both the lines

$$8\bar{x} - 10\bar{y} = -66$$

$$40\bar{x} - 18\bar{y} = 214$$

$\therefore$ Solving these two equations, we get

$$\bar{x} = 13, \quad \bar{y} = 17$$

(b) Let us assume that the line $8x - 10y + 66 = 0$ be the regression equation of y on x.

$$10y = 8x + 66 \Rightarrow y = \frac{8}{10}x + \frac{66}{10}$$

$$\Rightarrow b_{yx} = \frac{4}{5}$$

Also the regression equation of x on y

$$40x = 18y + 214 \Rightarrow x = \frac{18}{40}y + \frac{214}{48}$$

$$\Rightarrow b_{xy} = \frac{9}{20}$$

$$\therefore \quad r = \sqrt{b_{yx} b_{xy}} = \sqrt{\frac{4}{5} \times \frac{9}{20}} = \frac{6}{10}$$

(c) $\sigma_x^2 = 9 \Rightarrow \sigma_x = 3$

Also, $b_{yx} = r \dfrac{\sigma_y}{\sigma_x}$

$$\Rightarrow \frac{4}{5} = \frac{6}{10} \times \frac{\sigma_y}{3}$$

$$\Rightarrow \boxed{\sigma_y = 4}$$