Numerical Methods <u>Unit 5</u> 10/4/2024

2:00 pm - 4:00 pm.

Jacobi's iteration Method :-

$$\left.\begin{aligned} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{aligned}\right\} \text{——} (1)$$

If a_1, b_2, c_3 are large as compared to other coefficients, solve for x, y, z respectively.

$$\left.\begin{aligned} x &= \frac{1}{a_1}(d_1-b_1y-c_1z) \\ y &= \frac{1}{b_2}(d_2-a_2x-c_2z) \\ z &= \frac{1}{c_3}(d_3-a_3x-b_3y) \end{aligned}\right\} \text{——} (2)$$

Consider initial approximations as x_0, y_0, z_0 for x, y, z

$\therefore$ We substitute x_0, y_0, z_0 and get the 1st approximation

$$x_1 = \frac{1}{a_1}(d_1-b_1y_0-c_1z_0)$$

$$y_1 = \frac{1}{b_2}(d_2-a_2x_0-c_2z_0)$$

$$z_1 = \frac{1}{c_3}(d_3-a_3x_0-b_3y_0)$$

Substitute x_1, y_1, z_1 in (2), we get second approximation

$$x_2 = \frac{1}{a_1}(d_1-b_1y_1-c_1z_1)$$

$$y_2 = \frac{1}{b_2}(d_2-a_2x_1-c_2z_1)$$

$$z_2 = \frac{1}{c_3}(d_3-a_3x_1-b_3y_1)$$

This process is repeated till the difference betn two consecutive approximation is negligible.

Ex:- Solve, by Jacobi's iteration method, the equations

$$\left.\begin{aligned} 20x+y-2z&=17 \\ 3x+20y-z&=-18 \\ 2x-3y+20z&=25 \end{aligned}\right\} \text{ — } (1)$$

→ eqⁿˢ (1) we write as

$$x = \frac{1}{20}(17-y+2z)$$

$$y = \frac{1}{20}(-18-3x+z)$$

$$z = \frac{1}{20}(25-2x+3y)$$

$x_0=0,\ y_0=0,\ z_0=0$ Consider this initial approximation

∴ 1st approximation,

$$x_1 = \frac{1}{20}(17-y_0+2z_0) = \frac{17}{20} = 0.85$$

$$y_1 = \frac{1}{20}(-18-3x_0+z_0) = -\frac{18}{20} = -0.9$$

$$z_1 = \frac{1}{20}(25-2x_0+3y_0) = \frac{25}{20} = 1.25$$

2nd approximation,

$$x_2 = \frac{1}{20}(17-y_1+2z_1) = 1.02$$

$$y_2 = \frac{1}{20}(-18-3x_1+z_1) = -0.965$$

$$z_2 = \frac{1}{20}(25-2x_1+3y_1) = 1.03$$

3rd approximation,

$$x_3 = \frac{1}{20}(17-y_2+2z_2) = 1.00125$$

$$y_3 = \frac{1}{20}(-18-3x_2+z_2) = -1.0015$$

$$z_3 = \frac{1}{20}(25-2x_2+3y_2) = 1.00325$$

4th approximation, $x_4 = \frac{1}{20}(17-y_3+2z_3) = 1.0004$

$$y_4 = \frac{1}{20}(-18-3x_3+z_3) = -1.000025$$

$$z_4 = \frac{1}{20}(25-2x_3+3y_3) = 0.99965$$

5th approximation $x_5 = \frac{1}{20}(17 - y_4 + 2z_4) = 0.99996 6$

$y_5 = \frac{1}{20}(-18 - 3x_4 + z_4) = -1.000078$

$z_5 = \frac{1}{20}(25 - 2x_4 + 3y_4) = 0.999956$

6th approximation

$x_6 = 1.0000$

$y_6 = -0.999997$

$z_6 = 0.999992$

$x \cong 1,\ y \cong -1,\ z \cong 1$

Ex:- Solve by Jacobi's iteration method, the equations

$$10x + y - z = 11.19$$
$$x + 10y + z = 28.08$$
$$-x + y + 10z = 35.61$$

correct to two decimal places.

$\rightarrow$

$$x = \frac{1}{10}(11.19 - y + z)$$

$$y = \frac{1}{10}(28.08 - x - z)$$

$$z = \frac{1}{10}(35.61 + x - y)$$

$x_0, y_0, z_0 = 0$

$\therefore\ x_1 = \frac{1}{10}(11.19 - y_0 + z_0) = \frac{11.19}{10} = 1.19$

$y_1 = \frac{1}{10}(28.08 - x_0 - z_0) = \frac{28.08}{10} = 2.808$

$z_1 = \frac{1}{10}(35.61 + x_0 - y_0) = \frac{35.61}{10} = 3.561$

2nd iteration, $x_2 = \frac{1}{10}(11.19 - y_1 + z_1) = 1.19$

$y_2 = \frac{1}{10}(28.08 - x_1 - z_1) = 2.34$

$z_2 = \frac{1}{10}(35.61 + x_1 - y_1) = 3.39$

3rd iterations $x_3 = \frac{1}{10}(11.19 - y_2 + z_2) = 1.22$

$y_3 = \frac{1}{10}(28.08 - x_2 - z_2) = 2.35$

$z_3 = \frac{1}{10}(35.61 + x_2 - y_2) = 3.45$

4th iteration $x_4 = \frac{1}{10}(11.19 - y_3 + z_3) = 1.23$

$y_4 = \frac{1}{10}(28.08 - x_3 - z_3) = 2.34$

$z_4 = \frac{1}{10}(35.61 + x_3 - y_3) = 3.45$

5th iteration $x_5 = 1.23$

$y_5 = 2.34$

$z_5 = 3.45$

Hence $x = 1.23$, $y = 2.34$, $z = 3.45$.

Gauss-seidal method :-

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z)$$

$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z)$$

$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

x_0, y_0, z_0 $\quad x_1 = \frac{1}{a_1}(d_1 - b_1 y_0 - c_1 z_0)$

$$y_1 = \frac{1}{b_2}(d_2 - a_2 x_1 - c_2 z_0)$$

$$z_1 = \frac{1}{c_3}(d_3 - a_3 x_1 - b_3 y_1)$$

2nd iteration, $x_2 = \frac{1}{a_1}(d_1 - b_1 y_1 - c_1 z_1)$

$$y_2 = \frac{1}{b_2}(d_2 - a_2 x_2 - c_2 z_1)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3 x_2 - b_3 y_2)$$

& so on.

Ex:- $20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

→ $x = \frac{1}{20}(17 - y + 2z)$

$$y = \frac{1}{20}(-18 - 3x + z)$$

$$z = \frac{1}{20}(25 - 2x + 3y)$$

$x_0 = 0, y_0 = 0, z_0 = 0$

1st approx. $x_1 = \frac{1}{20}(17 - y_0 + 2z_0) = 0.8500$

$y_1 = \frac{1}{20}(-18 - 3x_1 + z_0) = -1.0275$

$z_1 = \frac{1}{20}(25 - 2x_1 + 3y_1) = 1.0109$

2nd approximation, $x_2 = \frac{1}{20}(17 - y_1 + 2z_1) = 1.0025$

$y_2 = \frac{1}{20}(-18 - 3x_2 + z_1) = -0.9998$

$z_2 = \frac{1}{20}(25 - 2x_2 + 3y_2) = 0.9998$

3rd iteration,

$x_3 = \frac{1}{20}(17 - y_2 + 2z_2) = 1.0000$

$y_3 = \frac{1}{20}(-18 - 3x_3 + z_2) = -1.0000$

$z_3 = \frac{1}{20}(25 - 2x_3 + 3y_3) = 1.0000$

Ex:- $4x + 2y + z = 14$

$x + 5y - z = 10$

$x + y + 8z = 20.$

$\rightarrow$ $x = \frac{1}{4}(14 - 2y - z)$

$y = \frac{1}{5}(10 - x + z)$

$z = \frac{1}{8}(20 - x - y)$

$x_0 = 0, y_0 = 0, z_0 = 0$

1st iteration, $x_1 = \frac{1}{4}(14 - 2y_0 - z_0) = \frac{14}{4} = 3.5$

$y_1 = \frac{1}{5}(10 - x_1 + z_0) = \frac{1}{5}(10 - 3.5) = 1.3$

$z_1 = \frac{1}{8}(20 - x_1 - y_1) = \frac{1}{8}(20 - 3.5 - 1.3) = 1.9$

2nd iteration, $x_2 = \frac{1}{4}(14 - 2y_1 - z_1) = \frac{1}{4}(14 - (2 \times 1.3) - 1.9) = 2.375$

$y_2 = \frac{1}{5}(10 - x_2 + z_1) = \cancel{1.8075}$ 1.905

$z_2 = \frac{1}{8}(20 - x_2 - y_2) = 1.965$

3rd iteration, $x_3 = \frac{1}{4}(14 - 2y_2 - z_2) = 2.07125$

$y_3 = \frac{1}{5}(10 - x_3 + z_2) = 1.98$

$z_3 = \frac{1}{8}(20 - x_3 - y_3) = 1.99$

4th iteration,

$x_4 = \frac{1}{4}(14 - 2y_3 - z_3) = 2.0125$

$y_4 = \frac{1}{5}(10 - x_4 + z_3) = 1.995$

$z_4 = \frac{1}{8}(20 - x_4 - y_4) = 1.999$

H.W :-

1. $28x + 4y - z = 32 \Rightarrow x =$
   $x + 3y + 10z = 24 \Rightarrow z =$
   $2x + 17y + 4z = 35 \Rightarrow y =$

2. $10x + y + z = 12$
   $2x + 10y + z = 13$
   $2x + 2y + 10z = 14$

3. $5x + 2y + z = 12$
   $x + 4y + 2z = 15$
   $x + 2y + 5z = 20$