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Q1. Define the following

a) Fourier Transform

→ The fourier Transform is defined as

$$F(\lambda) = \int_{-\infty}^{\infty} f(u) e^{-i\lambda u} du$$

b) Inverse Fourier Transform

→ The inverse fourier transform is defined as

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\lambda) e^{i\lambda x} d\lambda$$

c) Fourier sine transform

→ Fourier sine transform is defined as

$$F_s(\lambda) = \int_{0}^{\infty} f(u) \sin\lambda u \, du$$

d) Inverse fourier sine transform

→ Inverse fourier sine transform is

$$f(x) = \frac{2}{\pi} \int_{0}^{\infty} F_s(\lambda) \sin\lambda x \, d\lambda$$

e) Fourier cosine transform

→ The Fourier cosine transform is

$$F_c(\lambda) = \int_{0}^{\infty} f(u) \cos\lambda u \, du$$

f) Inverse Fourier cosine Transform

→ The inverse fourier cosine transform is

$$f(x) = \frac{2}{\pi} \int_{0}^{\infty} F_c(\lambda) \cos\lambda x \, d\lambda$$

Q2. Find the Fourier Transform of the following

a) $f(x) = \begin{cases} \sin 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$

Soln: Fourier Transform is given by,

$$F(\lambda) = \int_{-\infty}^{\infty} f(u) e^{-i\lambda u} du$$

$$f(u) = \begin{cases} \sin 2u, & u \geq 0 \\ 0, & u < 0 \end{cases}$$

$$F(\lambda) = \int_{0}^{\infty} f(u) e^{-i\lambda u} du + \int_{-\infty}^{-1} f(u) e^{-i\lambda u} du$$

$$= \int_{0}^{\infty} \sin 2u \, e^{-i\lambda u} du$$

$$= \left[\frac{e^{-i\lambda u}}{4-\lambda^2}[-i\lambda \sin 2u - 2\cos 2x]\right]_0^{\infty}$$

$F(\lambda) = \frac{2}{4-\lambda^2}$ is the fourier transform

b) $f(x) = \begin{cases} x^2, & x \geq 0 \\ 0, & x < 0 \end{cases}$

Soln: Fourier transform is given by,

$$F(\lambda) = \int_{-\infty}^{\infty} f(u) e^{-i\lambda u} du$$

$$f(u) = \begin{cases} u^2, & u \geq 0 \\ 0, & u < 0 \end{cases}$$

$$F(\lambda) = \int_{0}^{\infty} f(u) e^{-i\lambda u} du + \int_{-\infty}^{-1} f(u) e^{-i\lambda u} du$$

$$= \int_{0}^{\infty} u^2 e^{-i\lambda u} du$$

$$= \left[\frac{-u^2 e^{-i\lambda u}}{i\lambda} - \frac{2ue^{-i\lambda u}}{i^2\lambda^2} - \frac{2e^{-i\lambda u}}{i^3\lambda^3}\right]_0^{\infty}$$

$$= -\left[\frac{u^2 e^{-i\lambda u}}{i\lambda} - \frac{2ue^{-i\lambda u}}{\lambda^2} - \frac{2e^{-i\lambda u}}{i\lambda^3}\right]_0^{\infty}$$

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$F(\lambda) = \frac{-2}{i\lambda^3}$ is the fourier transform

c) $f(x) = \begin{cases} x+5 & , x \geq 0 \\ 0 & , x < 0 \end{cases}$

Soln: Fourier Transform is given by,

$$F(\lambda) = \int_{-\infty}^{\infty} f(u)\, e^{-i\lambda u}\, du$$

$$f(u) = \begin{cases} u+5, & u \geq 0 \\ 0, & u < 0 \end{cases}$$

$$F(\lambda) = \int_{0}^{\infty} f(u)\, e^{-i\lambda u}\, du + \int_{-\infty}^{-1} f(u) e^{-i\lambda u}\, du$$

$$= \int_{0}^{\infty} (u+5)\, e^{-i\lambda u}\, du$$

$$= \int_{0}^{\infty} u e^{-i\lambda u}\, du + \int_{0}^{\infty} 5 e^{-i\lambda u} du$$

$$= \left[\frac{-u e^{-i\lambda u}}{i\lambda} - \frac{e^{i\lambda u}}{i^2\lambda^2} \right]_0^{\infty} + 5\left[\frac{-e^{-i\lambda u}}{i\lambda} \right]_0^{\infty}$$

$$= \frac{-1}{\lambda^2} + \frac{5}{i\lambda}$$

Q3. Find the fourier sine transform of the following

a) $f(x) = \begin{cases} x & , 0 \leq x \leq 1 \\ 2-x & ; 1 \leq x \leq 2 \\ 0 & , x > 2 \end{cases}$

Soln: Fourier sine transform is given by,

$$F_s(\lambda) = \int_{0}^{\infty} f(u) \sin\lambda u\, du$$

$$f(u) = \begin{cases} u, & 0 \leq u \leq 1 \\ 2-u, & 1 \leq u \leq 2 \\ 0, & u > 2 \end{cases}$$

$$F_s(\lambda) = \int_0^1 f(u)\sin\lambda u\,du + \int_1^2 f(u)\sin\lambda u\,du$$

$$+ \int_2^\infty f(u)\sin\lambda u\,du$$

$$= \int_0^1 u\sin\lambda u\,du + \int_1^2 2\sin\lambda u\,du - \int_1^2 u\sin\lambda u\,du$$

$$= \left[-\frac{u\cos\lambda u}{\lambda} + \frac{\sin\lambda u}{\lambda^2}\right]_0^1 + 2\left[\frac{-\cos\lambda u}{\lambda}\right]_1^2$$

$$\left[\frac{u\cos\lambda u}{\lambda} - \frac{\sin\lambda u}{\lambda^2}\right]_1^2$$

$$= \frac{\sin\lambda}{\lambda^2} - \cancel{\frac{\cos\lambda}{\lambda}} + \frac{2\cos\lambda}{\lambda} - \cancel{\frac{2\cos 2\lambda}{\lambda}}$$

$$+ \cancel{\frac{2\cos 2\lambda}{\lambda}} - \frac{\sin 2\lambda}{\lambda^2} - \cancel{\frac{\cos\lambda}{\lambda}} + \frac{\sin\lambda}{\lambda^2}$$

$F_s(\lambda) = \dfrac{2\sin\lambda}{\lambda^2} - \dfrac{\sin 2\lambda}{\lambda^2}$ is the fourier sine

transform

v) $f(x) = \dfrac{e^{-ax}}{x}$

Solⁿ: Fourier sine transform is given by,

$$F_s(\lambda) = \int_0^\infty f(u)\sin\lambda u\,du$$

$$f(u) = \cancel{\frac{e^{-ax}}{x}}\ \frac{e^{-au}}{u}$$

$$F_s(\lambda) = \int_0^\infty \frac{e^{-au}}{u}\sin\lambda u\,du$$

$$\frac{dF_s(\lambda)}{d\lambda} = \int_0^\infty \frac{d}{d\lambda}\frac{e^{-au}}{u}\sin\lambda u\,du$$

$$= \int_0^\infty \frac{e^{-au}}{\cancel{u}} \times \cancel{u}\cos\lambda u\,du$$

$$= \int_0^\infty e^{-au} \cos \lambda u \, du$$

$$= \left[\frac{e^{-au}}{a^2+\lambda^2} (-a\cos\lambda u + \lambda \sin\lambda u) \right]_0^\infty$$

$$\frac{dF_s(\lambda)}{d\lambda} = \frac{a}{a^2+\lambda^2}$$

integrating,

$$F_s(\lambda) = \int \frac{a}{a^2+\lambda^2} d\lambda + C$$

$$F_s(\lambda) = \tan^{-1}\left(\frac{\lambda}{a}\right) + C \quad -①$$

put $\lambda = 0$ in ①,

$$\left[\int_0^\infty \frac{e^{-au}}{u} \sin\lambda u \, du \right]_{\lambda=0} = 0 + C$$

$$\therefore C = 0$$

$\therefore F_s(\lambda) = \tan^{-1}\left(\frac{\lambda}{a}\right)$ is the fourier sine transform

c) $f(x) = e^{-2x} + e^{-3x}$

Soln: Fourier sine transform is given by,

$$F_s(\lambda) = \int_0^\infty f(u) \sin\lambda u \, du$$

$$f(u) = e^{-2u} + e^{-3u}$$

$$F_s(\lambda) = \int_0^\infty e^{-2u} \sin\lambda u \, du + \int_0^\infty e^{-3u} \sin\lambda u \, du$$

$$= \left[\frac{e^{-2u}}{4+\lambda^2} [-2\sin\lambda u - \lambda\cos\lambda u] \right]_0^\infty$$

$$+ \left[\frac{e^{-3u}}{9+\lambda^2} (-3\sin\lambda u - \lambda\cos\lambda u) \right]_0^\infty$$

$F_s(\lambda) = \frac{\lambda}{4+\lambda^2} + \frac{\lambda}{9+\lambda^2}$ is the fourier sine transform.

Q4. Find the fourier cosine transform of the following

a) $f(x) = \begin{cases} x & , 0 \le x \le 1 \\ 2-x & , 1 \le x \le 2 \\ 0 & , x > 2 \end{cases}$

Soln: Fourier cosine transform is given by,

$$F_c(\lambda) = \int_0^{\infty} f(u) \cos\lambda u \, du$$

$$f(u) = \begin{cases} u & , 0 \le u \le 1 \\ 2-u & , 1 \le u \le 2 \\ 0 & , u > 2 \end{cases}$$

$$F_c(\lambda) = \int_0^1 u \cos\lambda u \, du + \int_1^2 2 \cos\lambda u \, du - \int_1^2 u \cos\lambda u \, du$$

$$= \left[\frac{u \sin\lambda u}{\lambda} + \frac{\cos\lambda u}{\lambda^2} \right]_0^1 + 2\left[\frac{\sin\lambda u}{\lambda} \right]_1^2$$

$$- \left[\frac{u \sin\lambda u}{\lambda} + \frac{\cos\lambda u}{\lambda^2} \right]_1^2$$

$$= \cancel{\frac{\sin\lambda}{\lambda}} + \frac{\cos\lambda}{\lambda^2} - \frac{1}{\lambda^2} + \cancel{\frac{2\sin 2\lambda}{\lambda}} - \cancel{\frac{2\sin\lambda}{\lambda}}$$

$$- \cancel{\frac{2\sin 2\lambda}{\lambda}} - \frac{\cos 2\lambda}{\lambda^2} + \cancel{\frac{\sin\lambda}{\lambda}} + \frac{\cos\lambda}{\lambda^2}$$

$F_c(\lambda) = \frac{2\cos\lambda}{\lambda^2} - \frac{\cos 2\lambda}{\lambda^2} - \frac{1}{\lambda^2}$ is the fourier cosine transform

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b) $f(x) = e^{-|x|}$ for all x

Solⁿ: $\therefore$ Fourier cosine transform is,

$$F_c(\lambda) = \int_0^\infty f(u) \cos\lambda u\,du$$

$$f(u) = \begin{cases} e^{-x}, & x \geq 0 \\ e^{+x}, & x < 0 \end{cases}$$

$$F_c(\lambda) = \int_0^\infty f(u) \cos\lambda u\,du$$

$$= \int_0^\infty e^{-u} \cos\lambda u\,du$$

$$= \left[\frac{e^{-u}}{1+\lambda^2}(\cos\lambda u + \lambda\sin\lambda u)\right]_0^\infty$$

$F_c(\lambda) = \frac{1}{1+\lambda^2}$ is the required fourier cosine transform

c) $f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$

Solⁿ: Fourier cosine transform is,

$$F_c(\lambda) = \int_0^\infty f(u) \cos\lambda u\,du$$

$$f(u) = \begin{cases} 1, & 0 \leq u \leq 1 \\ 0, & u > 1 \end{cases}$$

~~$F_c(\lambda) =$~~

$$F_c(\lambda) = \int_0^1 \cos\lambda u\,du$$

$$= \left[\frac{\sin\lambda u}{\lambda}\right]_0^1$$

$F_c(\lambda) = \frac{\sin\lambda}{\lambda}$ is the fourier cosine transform

d) $f(x) = e^{-2x} + e^{-3x}$

Soln: Fourier cosine transform is,

$$F_c(\lambda) = \int_0^\infty f(u)\cos\lambda u\, du$$

$$f(u) = e^{-2u} + e^{-3u}$$

$$F_c(\lambda) = \int_0^\infty e^{-2u}\cos\lambda u\, du + \int_0^\infty e^{-3u}\cos\lambda u\, du$$

$$= \left[\frac{e^{-2u}}{4+\lambda^2}(-2\cos\lambda u + \lambda\sin\lambda u)\right]_0^\infty$$

$$+ \left[\frac{e^{-3u}}{9+\lambda^2}(-3\cos\lambda u + \lambda\sin\lambda u)\right]_0^\infty$$

$$F_c(\lambda) = \frac{2}{4+\lambda^2} + \frac{3}{9+\lambda^2}$$ is the ~~fo~~ fourier cosine transform

Q5. Solve the following integral ~~fo~~ equations:

a) $\int_0^\infty f(x)\sin\lambda x\, dx = \begin{cases} 1-\lambda & , 0 \le \lambda \le 1 \\ 0 & , \lambda \ge 1 \end{cases}$

Soln: Fourier sine transform is given,

$$F(\lambda) = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1 \end{cases}$$

By inverse fourier sine transform,

$$f(x) = \frac{2}{\pi}\int_0^\infty F_s(\lambda)\sin\lambda x\, d\lambda$$

$$= \frac{2}{\pi}\int_0^1 \sin\lambda x\, d\lambda - \frac{2}{\pi}\int_0^1 \lambda\sin\lambda x\, d\lambda$$

$$= \frac{2}{\pi}\left[\frac{-\cos\lambda x}{x}\right]_0^1 - \frac{2}{\pi}\left[\frac{-\lambda\cos\lambda x}{x} + \frac{\sin\lambda x}{x^2}\right]_0^1$$

$$= \cancel{-\frac{2}{\pi}\frac{\cos x}{x}} + \frac{2}{\pi}\times\frac{1}{x} + \cancel{\frac{2}{\pi}\frac{\cos x}{x}} - \frac{2}{\pi}\frac{\sin x}{x^2}$$

$$f(x) = \frac{2}{\pi}\left[\frac{x - \sin x}{x^2}\right]$$

b) $\int_0^\infty f(x)\sin\lambda x \, dx = \begin{cases} 1, & 0 \le \lambda \le 1 \\ 2, & 1 \le \lambda \le 2 \\ 0, & \lambda \ge 2 \end{cases}$

Soln: Fourier sine transform is given,

$$F(\lambda) = \begin{cases} 1, & 0 \le \lambda \le 1 \\ 2, & 1 \le \lambda \le 2 \\ 0, & \lambda \ge 2 \end{cases}$$

By inverse fourier sine transform,

$$f(x) = \frac{2}{\pi}\int_0^\infty F_s(\lambda)\sin\lambda x \, d\lambda$$

$$= \frac{2}{\pi}\int_0^1 \sin\lambda x \, d\lambda + \frac{2}{\pi}\int_1^2 2\sin\lambda x \, d\lambda$$

$$= \frac{2}{\pi}\left[\frac{-\cos\lambda x}{x}\right]_0^1 + \frac{4}{\pi}\left[\frac{-\cos\lambda x}{x}\right]_1^2$$

$$= \frac{2}{\pi}\left[\frac{1}{x} - \frac{\cos x}{x}\right] + \frac{4}{\pi}\left[-\frac{\cos 2x}{x} + \frac{\cos x}{x}\right]$$

$$= \frac{2}{\pi}\frac{\cos x}{x} + \frac{2}{\pi}\frac{1}{x} - \frac{4}{\pi}\frac{\cos 2x}{x}$$

$$f(x) = \frac{2}{\pi}\left(\frac{\cos x}{x} + \frac{1}{x} - \frac{2\cos 2x}{x}\right)$$

c) $\int_0^\infty f(x)\cos\lambda x \, dx = e^{-\lambda}, \lambda > 0$

Soln: Fourier cosine transform is given,

$$F_c(\lambda) = e^{-\lambda}, \lambda > 0$$

By inverse fourier cosine transform,

$$f(x) = \frac{2}{\pi}\int_0^\infty F_c(\lambda)\cos\lambda x \, d\lambda$$

$$= \frac{2}{\pi}\int_0^\infty e^{-\lambda}\cos\lambda x \, d\lambda$$

$$= \frac{2}{\pi}\left[\frac{e^{-\lambda}}{1+x^2}\left[-\cos\lambda x + x\sin\lambda x\right]\right]_0^\infty$$

$$f(x) = \frac{2}{\pi}\left(\frac{1}{1+x^2}\right)$$

d) $\int_0^\infty f(x)\cos\lambda x\, dx = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1 \end{cases}$

Soln: Fourier cosine transform is given,

$$F_c(\lambda) = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1 \end{cases}$$

By inverse fourier cosine transform,

$$f(x) = \frac{2}{\pi}\int_0^\infty F_c(\lambda)\cos\lambda x\, d\lambda$$

$$= \frac{2}{\pi}\int_0^1 \cos\lambda x\, d\lambda - \frac{2}{\pi}\int_0^1 \lambda\cos\lambda x\, d\lambda$$

$$= \frac{2}{\pi}\left[\frac{\sin\lambda x}{x}\right]_0^1 - \frac{2}{\pi}\left[\frac{\lambda\sin\lambda x}{x} + \frac{\cos\lambda x}{x^2}\right]_0^1$$

$$= \cancel{\frac{2}{\pi}\frac{\sin x}{x}} - \cancel{\frac{2}{\pi}\frac{\sin x}{x}} - \frac{2}{\pi}\frac{\cos x}{x^2} + \frac{2}{\pi}\frac{1}{x^2}$$

$$f(x) = \frac{2}{\pi}\left[\frac{1-\cos x}{x^2}\right]$$ ~~The~~

Q6. Write short note on Discrete and Fast Fourier Transform.

Soln. The Discrete Fourier Transform (DFT) is a mathematical technique used to analyze the frequency content of a discrete signal.

- It transforms a sequence of complex numbers, representing the samples of a signal, into another sequence of complex numbers representing the signal in the frequency domain.
- The Fast Fourier Transform (FFT) is an

algorithm used to compute DFT efficiently.

- It reduces the number of computations required ~~from~~ for the DFT from $O(n^2)$ to $O(n \log n)$, n being the number of samples in the signal.

Q7. Write applications of Fourier Transform in different fields of Engineering.

Soln: Following are applications of fourier transform:-

1. Signal Processing: Enhancing communication
2. Image processing: Sharpening Visuals
3. control systems: Ensuring Stability
4. mechanical engineering: Vibrations and Acoustics
5. Electrical engineering: Power Systems Analysis
6. Structural engineering: Identifying Weaknesses