22/4/2024.

Curve Fitting :-

$(x_i, y_i) \rightarrow y \cong P_n(x)$ — Lagrange's / Newton interpolation

(x_i, y_i) $\quad y = a_0 x^n + a_1 x^{n-1} + \cdots + a_n$

Least square method :-

Straight line :- $\quad y = ax + b$

(x_1, y_1) (x_2, y_2) $\cdots$ (x_n, y_n)

Observed values $\quad y_1' = ax_1 + b$, $y_2' = ax_2 + b$, $\cdots$, $y_n' = ax_n + b$

$S = \text{Error} = (y_1' - y_1)^2 + (y_2' - y_2)^2 + \cdots + (y_n' - y_n)^2$

$S = e_1^2 + e_2^2 + \cdots + e_n^2$

$S = (ax_1 + b - y_1)^2 + (ax_2 + b - y_2)^2 + \cdots + (ax_n + b - y_n)^2$

$S = \text{fun}(a, b)$

$\frac{\partial S}{\partial b} = 0$, $\frac{\partial S}{\partial a} = 0$

$\frac{\partial S}{\partial b} = 2(ax_1 + b - y_1) + 2(ax_2 + b - y_2) + \cdots + 2(ax_n + b - y_n) = 0$

$a(x_1 + x_2 + \cdots + x_n) + nb - (y_1 + y_2 + \cdots + y_n) = 0$

$\therefore \quad a\sum x + nb = \sum y$ —— ①

$\frac{\partial S}{\partial a} = 2(ax_1 + b - y_1)x_1 + 2(ax_2 + b - y_2)x_2 + \cdots + 2(ax_n + b - y_n)x_n = 0$

$\therefore \; a(x_1^2 + x_2^2 + \cdots + x_n^2) + b(x_1 + x_2 + \cdots + x_n) - (x_1y_1 + x_2y_2 + \cdots + x_ny_n) = 0$

$\therefore \quad a\sum x^2 + b\sum x = \sum xy$ —— ②

$$y = ax + b$$

$$\sum y = a\sum x + nb \quad \text{——} ①$$

$$\sum xy = a\sum x^2 + b\sum x \quad \text{——} ②$$

Ex:- Fit a straight line of the form $y = mx + c$ to the following data, by using the method of least square.

x	y	xy	x^2
0	-5	0	0
1	-3	-3	1
2	-1	-2	4
3	1	3	9
4	3	12	16
5	5	25	25
6	7	42	36
7	9	63	49
28	16	140	140

$y = mx + c$ $\qquad n = 8$

$\therefore \sum y = m\sum x + nc$

$\therefore 16 = 28m + 8c \quad \text{——} ①$

$\sum xy = m\sum x^2 + c\sum x$

$\therefore 140 = 140m + 28c \quad \text{——} ②$

$m = 2$

$c = -5$

$$\boxed{\therefore y = 2x - 5}$$

Fitting a parabola :- $y = ax^2 + bx + c$

$$\sum y = a\sum x^2 + b\sum x + nc \quad \text{——} ①$$

$$\sum xy = a\sum x^3 + b\sum x^2 + c\sum x \quad \text{——} ②$$

$$\sum x^2 y = a\sum x^4 + b\sum x^3 + c\sum x^2 \quad \text{——} ③$$

Ex 2:- Fit a parabola of the form $y = ax^2 + bx + c$ to the following data

x	y	x^2	x^3	x^4	xy	x^2y
1	-5	1				
2	-2	4				
3	5	9				
4	16	16				
5	31	25				
6	50	36				
7	73	49				

$\sum x = 28$

$\sum y = 168$

$\sum x^2 = 140$

$\sum x^3 = 784$

$\sum x^4 = 4676$

$\sum xy = 1036$

$\Sigma x^2 y = 6440$

$y = ax^2 + bx + c$ $\qquad n = 7$

$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$ $\qquad \therefore 168 = 140a + 28b + 7$

$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$ $\qquad \therefore 1036 = a(784) + 140b + c(28)$

$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$ $\qquad \therefore 6440 = a(4676) + b(784) + c(140)$

$a = 2,\ b = -3,\ c = -4$

$y = 2x^2 - 3x - 4.$

Ex:- Fit a straight line of the form $y = a + bx$ to the following data by using method of least square

x	y	x^2	xy
0	12		
5	15		
10	17		
15	22		
20	24		
25	30		

$y = a + bx$

$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$\Sigma x^2 = 1375$

$\Sigma x = 75,\quad \Sigma y = 120,\ \Sigma xy = 1805$

$a = \qquad b =$

$$\left.\begin{aligned}120 &= 6a + 75b\\ 1805 &= 75a + 1375b\end{aligned}\right\} \Rightarrow \quad \begin{aligned}a &= 11{\cdot}28\\ b &= 0{\cdot}69\end{aligned}$$

$y = 11{\cdot}28 + 0{\cdot}69x$

Fit a curve $y = ax^b$

$\log y = \log a + b\log x$

$Y = \log a + bX = bX + C$

$Y = \log y,\ X = \log x,\ C = \log a.$

$$Y = bX + C \Rightarrow \sum Y = b\sum X + nC$$
$$\sum XY = b\sum X^2 + C\sum X$$

Ex:- Fit a curve $y = ax^b$ using the following data

x	y	$X = \log x$	$Y = \log y$	X^2	XY
2000	15	3.30103	1.17609	10.8967	3.88498
3000	15.5	3.47712	1.19033	12.09037	4.1389
4000	16	3.60206	1.20412	12.97483	4.3373
5000	17	3.69897	1.23044	13.6823	4.5513
6000	18	3.77815	1.2552	14.2744	4.7426
		17.85	6.056	63.9188	21.652

$$6.056 = 17.85b + 5c$$
$$21.625 = 63.9188b + 17.85c$$
$$b = 0.026, \quad c = 1.172$$

$$c = \log a \quad \therefore a = e^c = 13.09$$

$$y = ax^b$$
$$= 13.09\, x^{0.026}$$

Tut :- Fit a least square curve $y = ax^b$ to the following data

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

2. Fit a parabola $y = a + bx + cx^2$ to the following data

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

Recall :-

1. Mean

2. Std. deviation (σ), Mean deviation

$$\text{Mean deviation} = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n} \quad \text{OR} \quad \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{N}$$

$N = \sum f_i$

$$\text{Std. deviation } (\sigma) = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} \quad \text{OR} \quad \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum f x^2}{N} - \left(\frac{\sum f x}{N}\right)^2}$$ (if freq. distribution data)

$U = x - A$ or $\boxed{U = \frac{x - A}{h}} \rightarrow \sigma = h\sqrt{\frac{\sum f u^2}{N} - \left(\frac{\sum f u}{N}\right)^2}$

$\sigma_x = \sigma_u$

3) Variance = $(\text{Std. deviation})^2 = \sigma^2$

4) Coefficient of variance (C.V) $= \frac{\sigma}{\bar{x}} \times 100$

C.V of x < C.V of y $\Rightarrow$ x is more consistent than y

C.V of x > C.V of y $\Rightarrow$ x is more variable than y.

5) Correlation coeff $r(x, y) = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$

$\text{cov}(x, y) = \frac{\sum xy}{n} - \bar{x}\bar{y}$ = covariance of x & y.

Note: $-1 \leq r \leq 1$

For calculation, $r(x, y) = r(u, v)$, $U = x - A$, $V = y - B$.

6) Regression lines

y on x

$$y-\bar{y} = r\frac{\sigma_y}{\sigma_x}(x-\bar{x}) = b_{yx}(x-\bar{x})$$

x on y

$$x-\bar{x} = r\frac{\sigma_x}{\sigma_y}(y-\bar{y}) = b_{xy}(y-\bar{y})$$

$$\left.\begin{array}{l} b_{yx} = r\dfrac{\sigma_y}{\sigma_x} \\ b_{xy} = r\dfrac{\sigma_x}{\sigma_y} \end{array}\right\} \Rightarrow r = \pm\sqrt{b_{yx}\, b_{xy}}$$

7) Curve fitting using least square method

$y = ax+b$

$$\left.\begin{array}{l} \Sigma y = a\Sigma x + nb \\ \Sigma xy = a\Sigma x^2 + b\Sigma x \end{array}\right\} \Rightarrow \begin{array}{l} a = \\ b = \end{array}$$

$y = ax^2+bx+c$

$$\left.\begin{array}{l} \Sigma y = a\Sigma x^2 + b\Sigma x + nc \\ \Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x \\ \Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 \end{array}\right\} \Rightarrow \begin{array}{l} a = \\ b = \\ c = \end{array}$$

$y = ax^b \Rightarrow \log y = \log a + b\log x$

$$Y = C + bX$$

$$\therefore \Sigma Y = nC + b\Sigma X$$

$$\Sigma XY = C\Sigma X + b\Sigma X^2$$

b, C $\qquad C = \log a \quad \therefore a = e^C$