Unit 4 Probability

Modern Education Society's Wadia College Of Engineering, Pune-01
S.E(Computer)

207003: Engineering Mathematics III

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1. What is the probability of getting tail in a throw of a coin?

> **Solution:** When we toss a coin, there are two possible outcomes, viz., Head or Tail.
>
> $$\begin{aligned} \text{In this case the number of possible cases} &= n = 2 \\ \text{Number of favourable cases} &= m = 1. \\ \therefore \text{Probabilty of getting a tail} &= \frac{m}{n} = \frac{1}{2} \end{aligned}$$

2. A bag contains 6 white balls,9 black balls. What is the probabilty of drawing a black ball?

> **Solution:**
>
> $$\begin{aligned} \text{Total number of favourable events} &= n = 6 + 9 = 15 \\ \text{Number of favourable cases} &= m = 9. \\ \therefore \text{Probabilty of drawing a black ball} &= \frac{m}{n} = \frac{9}{15} = \frac{3}{5} \end{aligned}$$

3. What is the probabilty when a card is drawn at random from an ordinary pack of cards, if it is
   (a) a red card
   (b) a club

> **Solution:** Number of exhaustive cases =52
>
> (a) There are 26 red cards and 26 black cards in an ordinary pack.
>
> $$\begin{aligned} \therefore \text{Number of favourable cases} &= m = 26 \\ \therefore \text{Probabilty of getting a red card ball} &= \frac{m}{n} = \frac{26}{52} = \frac{1}{2} \end{aligned}$$

(b)

$$\therefore \text{Number of favourable cases} = m = 13$$
$$\therefore \text{Probabilty of getting a red card ball} = \frac{m}{n} = \frac{13}{52} = \frac{1}{4}$$

4. What is the probabilty that a leap year, selected at random, will have 53 Sundays?

Solution: There are 366 days in a leap year and it has 52 complete weeks and 2 days over. These two extra days can occur in the following possible ways

1. Sunday and Monday
2. Monday and Tuesday
3. Tuesday and Wednesday
4. Wednesday and Thursday
5. Thursday and Friday
6. Friday and Saturday
7. Saturday and Sunday

$$\text{Total number of favourable cases} = n = 7$$
$$\therefore \text{Number of favourable cases} = m = 2$$
$$\therefore \text{Probabilty of getting a red card ball} = \frac{m}{n} = \frac{2}{7}$$

Theroems on probability

- **Addition Thorem or Thorem on Total Probability:** If n events are mutually exclusive, then the probabilty of happening of any one of them is equal to the sum of the probabilities of the happening of the separate events. If $E_1, E_2, \cdots E_n$ be n events and $P(E_1), P(E_2), \cdots P(E_n)$, be their respective prbabilities, then

$$P(E_1 + E_2 + \cdots + E_n) = P(E_1) + P(E_2) + \cdots + P(E_n)$$

- **Multiplication Thorem or Thorem on Compound Probability:** The probabilty of the simulatneous occurence of the two events A and B is equal to the probabilty of one of the events multiplied by the conditional probability of the other, given occurence of the first,i.e

$$P(AB) = P(A)P(B|A) = P(B)P(A|B)$$

1. If the probabilty of the horse A winning the race is $\frac{1}{5}$ and the horse B winning the race is $\frac{1}{6}$, what is the probabilty that one of the horses will win the race?

> **Solution:**
> $$\text{Probabilty of winning of the horse A} = \frac{1}{5}$$
> $$\text{Probabilty of winning of the horse B} = \frac{1}{6}$$
> $$\therefore P(A+B) = P(A) + P(B) = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$$

2. A card is drawn from a pack of 52 cards and then a second card is drawn. What is the probabilty that both the cards drawn are queen?

> **Solution: First draw**:Probabilty of getting a queen=$\frac{4}{52}$=$\frac{1}{13}$
> **Second draw:** After drawing the first queen, we are left with 51 cards having 3 queens.
> $$\text{Probabilty of a getting a queen in second draw} = \frac{3}{51} = \frac{1}{17}$$
> $$\therefore \text{probabilty that both the cards drawn are queen} = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$$

Complementary events: The event 'A occurs' and the evnt 'A does not occur' are called complementary events.

$$P(A) + P(A^c) = 1$$

1. A problem in Mathematics is given to three students A,B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probabilty that the problem get solved?

> **Solution:** The probabilites of A,B and C solving the problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively.
> $$\therefore \text{The probabilites of all three do not solve the problem} = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$
> $$\text{The probabilty that the problem will be solved by atleast one of them} = 1 - \frac{1}{4} = \frac{3}{4}$$

Addition Theorem for compatible events: The probabilty of the occurrence of at least one of the events A and B (not mutually exclusive) is given by

$$P(A+B) = P(A) + P(B) - P(AB)$$

It is also written as

$$P(A\,\text{or}B) = P(A) + P(B) - P(A\,\text{and}\ B)$$

1. The probabilty that a student passes a Physics test is $\frac{2}{3}$ and probabilty that he passes both Physics and English test is $\frac{14}{45}$. The probabilty that he passes at least one test is $\frac{4}{5}$. What is the probabilty that the student passes the English test.

Solution: Let A denote that the student passes the Physics test:

$$\therefore P(A) = \frac{2}{3}$$

Let B denote that the student passes the English test:

$$P(B) = ?$$

It is given that the probabilty that the studnet passes both Physics and English test is $\frac{14}{45}$. Also the probabilty that he passes at least one test is $\frac{4}{5}$

$$\therefore P(A\text{or}B) = \frac{4}{5}$$

$$P(A\,\text{or}B) = P(A) + P(B) - P(A\,\text{and}\ B)$$
$$\frac{4}{5} = \frac{2}{3} + P(B) - \frac{14}{45}$$
$$\therefore P(B) = \frac{4}{9}$$