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UNIT : NO : 5 : (FIVE) :-

01/04/24 MON.

MATHEMATICAL METHODS.

SOLUTION OF ALGEBRAIC OR TRANSCENDENTAL EQUATIONS:-

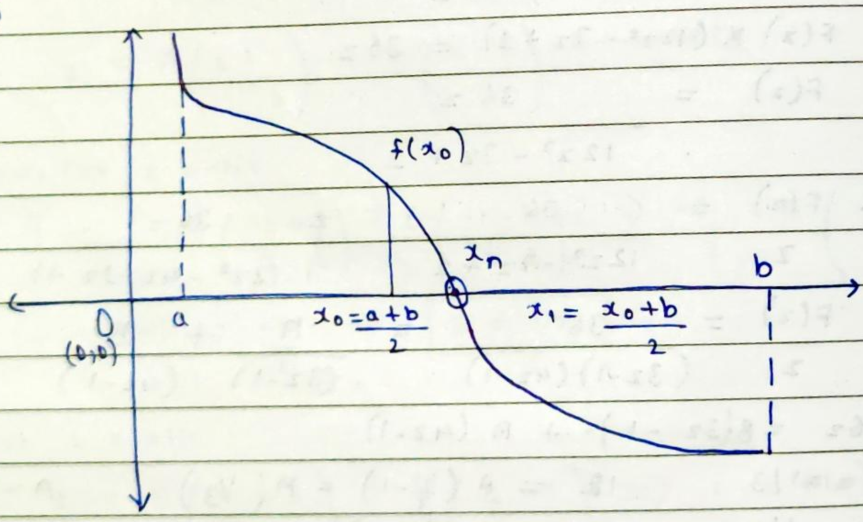
1. Bisection Method.
2. Regula - Falsi $\boxed{\text{OR}}$ False position. Method.
3. Secant Method.
4. Newton Raphson Method.

(1.) **BISECTION - METHOD:-**

$f(x) = 0$.

A value ~~of~~ 'α' of x which satisfies the given Equation (2) is called a root of Equation (1)

⇒



Q.] Find a root of Equation $x^3 - 4x - 9 = 0$, using the bisection Method correct it to three decimal places.

ANS. Given:- $f(x) = x^3 - 4x - 9$

Now, $x^3 - 4x - 9 = 0$.

$\therefore \boxed{x = 1, 2, 3}$ .... (Roots)

Now, substitute the roots in the function $F(x)$

$F(1) = 1 - 4 - 9 = 1 - 13 = -12 < 0$ $\quad \boxed{f(1) = -12 < 0}$

✓ $f(2) = \cancel{8} - 8 - 9 = -9 < 0$ $\quad \boxed{f(2) = -9 < 0}$

✓ $F(3) = 27 - 12 - 9 = 6 > 0$ $\quad \boxed{f(3) = 6 > 0}$

∴ The Root lies between '2' and '3'

$$\therefore x_0 = \frac{2+3}{2} = \frac{5}{2} = 2.5 \qquad \boxed{x_0 = 2.5}$$

$$f(2.5) = (2.5)^3 - 4(2.5) - 9$$

$$\boxed{f(2.5) = -3.375}$$

Now, The Root lies between '2.5' and '3' :-

$$x_1 = \frac{2.5+3}{2} = \frac{5.5}{2} = 2.75 \qquad \boxed{x_1 = 2.75}$$

$$f(2.75) = (2.75)^3 - 4(2.75) - 9 = 0.796875 > 0$$

∴ Root lies between 2.75 and 2.75

$$x_2 = \frac{2.5 + 2.75}{2} = 2.625 \qquad \boxed{x_2 = 2.625}$$

Now, $f(2.625) = (2.625)^3 - 4(2.625) - 9$

$$= \underline{-1.4121} < 0.$$

Root lies between 2.75 and 2.625

$$x_3 = \frac{2.75 + 2.625}{2} = 2.6875$$

$$f(2.6875) = (2.6875)^3 - 4(2.6875) - 9$$

$$= \underline{-0.33} < 0.$$

Root lies between 2.75 and 2.6875

$$x_4 = \frac{2.75 + 2.6875}{2} = 2.71875$$

$$f(2.71875) = (2.71875)^3 - 4(2.71875) - 9$$

$$= \underline{0.220} > 0$$

Root lies between 2.71875 and 2.6875

$$x_5 = \frac{2.71875 + 2.6875}{2} = \underline{2.703125}$$

$$f(2.703125) = (2.703125)^3 - 4(2.703125) - 9$$

$$\boxed{f(2.703125) = -\underline{0.0610} < 0}$$

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Root lies between 2.703125 and 2.71875

$\therefore x_6 = \frac{2.703125 + 2.71875}{2} = 2.7109325$

$f(2.7109325) = (2.7109325)^3 - 4(2.7109325) - 9$

$= \underline{0.0805} > 0$

Root lies between 2.711 and 2.703

$\therefore x_7 = \frac{2.711 + 2.703}{2} = \underline{2.707}$

$f(2.707) = 8.48 \times 10^{-3} > 0$

Root lies between 2.707 and 2.703

$\therefore x_8 = \frac{2.707 + 2.703}{2} = \underline{2.705}$

$f(2.705) = -0.027 < 0$

Root lies between 2.705 and 2.707

$\therefore x_9 = \frac{2.705 + 2.707}{2} = 2.706.$

$f(2.706) = -9.48 \times 10^{-3} < 0$

Root lies between 2.706 and 2.707

$\therefore x_{10} = \frac{2.706 + 2.707}{2} = 2.70675$

$f(2.7065) = -5.025 \times 10^{-4} < 0$

Root lies between 2.7065 and 2.707

$\therefore x_{11} = \frac{2.7065 + 2.707}{2} = 2.70675$

$f(2.70675) = 3.99 \times 10^{-3} > 0$

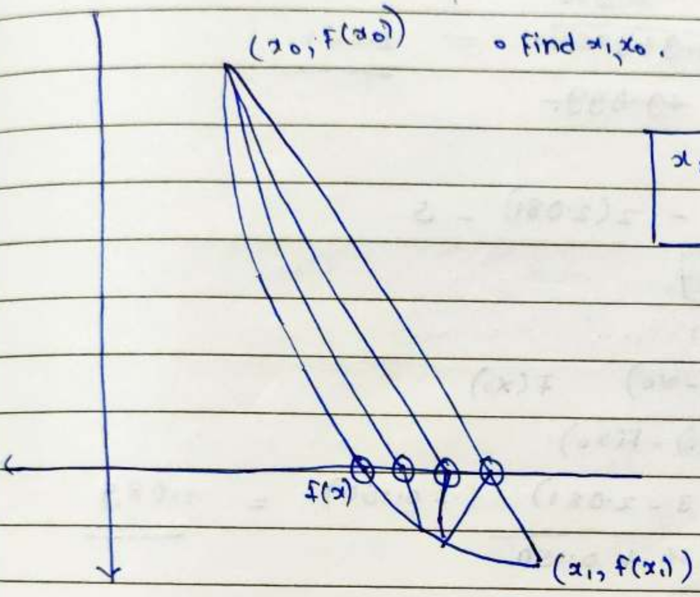
Root lies between 2.70675 and 2.7065

$\therefore x_{12} = \dfrac{2.70675 + 2.7065}{2} = 2.706$

$\boxed{x_{12} = 2.706}$

(2) FALSE POSITION OR REGULA-FALSI METHOD

$\rightarrow$



- find x_1, x_0.

$y - f(x_0) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$

$\boxed{x_2 = x_0 \mp \dfrac{x_1 - x_0}{f(x_1 - x_0)}(x - x_0)}$

Q. Find a real root of the Equation $x^3 - 2x - 5 = 0$ by method of false position. Correct it to 3 decimal places.

$\rightarrow$ $f(x) = x^3 - 2x - 5$

$f(2) = -1 < 0$

$f(3) = 16 > 0.$

$\therefore$ Root lies between 2 and 3.

$x_0 = 2 \quad x_1 = 3$

$\therefore x_2 = x_0 \mp \dfrac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0)$

$x_2 = 2 - \dfrac{(3-2)}{16+1}(-1) = \dfrac{2+1}{17} = 2.058$

$\boxed{x_2 = 2.058}$

$f(x_2) = (2.058)^3 - 2(2.058) - 5 = -0.399$

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Choose $x_0 = 2.058$

$x_1 = 3$

$$x_3 = x_0 - \frac{x_1 - x_0}{F(x_1) - F(x_0)} \cdot F(x_0)$$

$$\therefore x_3 = 2.058 - \frac{(3 - 2.058)}{16 + 3.999} (-3.999)$$

$$\therefore x_3 = \cancel{2.058 + \frac{2.919258}{19.099}} = \underline{2.081}$$

$\boxed{x_3 = 2.081}$

$F(x_3) = (2.081)^3 - 2(2.081) - 5$

$\boxed{F(x_3) = -0.150}$

$$x_4 = x_0 - \frac{(x_1 - x_0)}{F(x_1) - F(x_0)} F(x_0)$$

$$x_4 = 2.081 - \frac{(3 - 2.081)}{16 + 0.150} (-0.150) = \underline{2.089}$$

$x_4 = 2.089$

$f(x_4) = (2.089)^3 - 2(2.089) - 5$

$\boxed{F(x_4) = -0.0617}$

$$x_5 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \cdot f(x_0)$$

$$\therefore x_5 = 2.089 - \frac{(3 - 2.089)}{16 + 0.0617} (-0.0617)$$

$$\therefore x_5 = 2.089 + \frac{0.05620}{16.0617} = 2.0924$$

$F(x_5) = (2.092)^3 - 2(2.092) - 5$

$F(x_5) = -0.028$

$$x_6 = x_0 - \frac{(x_1 - x_0)}{F(x_1) - F(x_0)} \cdot f(x_0)$$

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$$x_6 = 2.092 - \frac{(3 - 2.092)}{16 + 0.028}(-0.028)$$

$$\therefore x_6 = 2.092 + (1.586 \times 10^{-3})$$

$$\therefore x_6 = 2.0935 \approx 2.094$$

$$\boxed{x_6 = 2.094}$$

$$\boxed{x_6 = 2.094}$$

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(3) SECANT - METHOD:-

→ Choose x_0, x_1 s.t.

$f(x_0) \cdot f(x_1) < 0$

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0)$$

$$x_3 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \cdot f(x_1)$$

$$x_4 = x_2 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} \cdot f(x_2)$$

⋮

$$\boxed{x_n = x_{n-2} - \frac{x_{n-1} + x_{n-2}}{f(x_{n-1}) + f(x_{n-2})} \cdot f(x_{n-2})}$$

Q.) Find a Real root of Equation $x^3 - 2x - 5 = 0$ by secant Method. Correct it up to 3 decimal places.

ANS. SOLUTION:-

$f(x) = x^3 - 2x - 5 = 0.$

$f(2) = -1 < 0$

$f(3) = 16 > 0$

$x_0 = 2, \quad x_1 = 3$

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0)$$

$$\therefore x_2 = 2 - \frac{3 - 2}{16 + 1}(-1) = 2 + \frac{1}{17} = 2.058$$

$f(x_2) = (2.058)^3 - 2(2.058) - 5 = -0.399$

$$\therefore x_3 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \cdot f(x_1)$$

$$\therefore x_3 = 3 - \frac{(2.058 - 3)}{-0.399 - 16} \cdot (16) = 2.080.$$

$$\boxed{x_3 = 2.080}$$

$f(x_3) = (2.080)^3 - 2(2.080) - 5$

$f(x_3) = -0.150 < 0$

$$x_4 = x_2 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_2)$$

$$\therefore x_4 = 2.058 - \frac{(2.081 - 2.058)}{-0.151 + 0.399} \cdot (-0.399)$$

$$\therefore x_4 = 2.058 + \frac{9.177 \times 10^{-3}}{0.248} = 2.095$$

$f(x_4) = 2.095^3 - (2 \times 2.095) - 5$

$= 5.0073 \times 10^{-3}$

$$x_5 = x_3 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_3)$$

$$x_5 = 2.081 - \frac{(2.095 - 2.081)}{5.0073 \times 10^{-3} + 0.151} (-0.151)$$

$$x_5 = 2.081 + \frac{2.114 \times 10^{-3}}{0.1560073} = 2.09455$$

$$\boxed{x_5 = 2.094}$$

Q) $\cos x = x \cdot e^x$

ANS. SOLUTION:- $\cos x - x \cdot e^x = 0.$

$f(1) = \cos(1) - 1 \cdot e^1 = -2.177 < 0$

$f(2) = \cos 2 - 2 \cdot e^2 = -15.194 < 0$

$f(3) = \cos 3 - 3 \cdot e^3 = -61.2466 < 0$

$f(0) = \cos 0 - 0 \cdot e^0 = 1 - 0 = 1$

$x_0 = 0 \qquad x_1 = 1$

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0) = 0 - \frac{(1-0)}{-2.177 - 1}(1)$$

$$\therefore x_2 = 0 + \frac{1}{3.177} = 0.31467. \qquad \boxed{x_2 = 0.31467}$$

$$F(x_2) = \cancel{(0.31467)^3 - 2(0.31467) - 5}$$

$$= \cos(0.31467) - (0.31467) \cdot e^{0.31467}$$

$$= 0.51986$$

$$x_3 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \cdot f(x_1)$$

$$\therefore x_3 = 1 - \frac{(0.31467 - 1)}{0.51986 + 2.177} \cdot (-2.177) = 1 + \frac{(0.31467 - 1)(2.177)}{0.51986 + 2.177}$$

$$\therefore \boxed{x_3 = \cancel{0.7458}\ \cancel{.4467}} \qquad \boxed{x_3 = 0.44673}$$

$$f(x_3) = \cancel{(0.7458)^3 - 2} \cos(0.4467) - (0.4467) \cdot e^{0.4467}$$

$$\boxed{f(x_3) = \cancel{-0.83769}} \ \cancel{0.2036} \qquad \boxed{f(x_3) = 0.20354}$$

$$x_4 = x_2 - \frac{(x_3 - x_2)}{f(x_3) - f(x_2)} \times f(x_2)$$

$$\therefore x_4 = 0.31467 - \frac{(0.44673 - 0.31467)(0.51986)}{(0.20354 - 0.51986)}$$

$$\therefore x_4 = 0.31467 + 0.21703 = 0.53171$$

$$\boxed{x_4 = 0.53171}$$

$$f(x_4) = \cos(0.53171) - (0.53171 e^{0.53171})$$

$$\boxed{f(x_4) = -0.04294}$$

$$x_5 = x_3 - \frac{(x_4 - x_3)}{f(x_4) - f(x_3)} \cdot f(x_3)$$

$$\therefore x_5 = 0.44673 - \frac{(0.53171 - 0.44673) \times (0.20354)}{(-0.04294 - 0.20354)}$$

$$\therefore \boxed{x_5 = \cancel{0.55443}\ 0.51690}$$

$$f(x_5) = \cos(0.51690) - (0.51690 e^{0.51690})$$

$$\therefore f(x_5) = 0.00260 \approx 2.6063 \times 10^{-3}$$

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$$x_6 = x_4 - \frac{(x_5 - x_4)}{f(x_5) - f(x_4)} \times f(x_4)$$

$$\therefore x_6 = 0.53171 - \frac{(0.51690 - 0.53171)}{(0.00260 + 0.04294)} (-0.04294)$$

$$\therefore x_6 = 0.53171 - 0.01396$$

$$\therefore \boxed{x_6 = 0.51775}$$

$$f(x_6) = \cos(0.51775) - (0.51775 e^{0.51775}) = 0.000034$$

$$x_7 = x_5 - \frac{(x_6 - x_5)}{f(x_6) - f(x_5)} \times f(x_5)$$

$$x_7 = 0.51690 - \frac{(0.51775 - 0.51690)}{(0.000034 - 0.00260)} \times (0.00260)$$

$$x_7 = 0.51690 - (-0.00086) = 0.51776$$

$$\boxed{x_7 = 0.51776}$$

Here $\boxed{x_6 \approx x_7}$

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(4.) NEWTON RAPHSON METHOD:-

→ Let x_0 be approximate root of $f(x) = 0$.

$x_1 = x_0 + h$ be the exact root.

$f(x_1) = 0$

$f(x_0 + h) = 0$.

Taylor's series,

$$f(x_0) + h \cdot f'(x_0) + \underbrace{\frac{h^2}{2!} \cdot f''(x_0) + \dots\dots + }_{\text{Neglect it.}} = 0.$$

$$f(x_0) + h \cdot f'(x_0) = 0.$$

$$\therefore \quad h = -\frac{f(x_0)}{f'(x_0)}$$

$$\therefore \boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$$\vdots$$

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

$$\vdots$$

$$\boxed{x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}}$$

Q.) $\cos x = x \cdot e^x$.

ANS. -SOLUTION:- $f(x) = \cos x - x \cdot e^x$

$f'(x) = -\sin x - (x \cdot e^x + e^x)$

Now, The Root lies between 0 and 1

$\therefore x_0 = 0.5$

By Newton Raphson Method ; we get.

$f(0.5) = 0.0532$

$f'(0.5) = -2.952$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = 0.5 - \frac{0.0532}{-2.952} = 0.5 + \frac{0.0532}{2.952}$$

$$\therefore \boxed{x_1 = 0.5180}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f(x_1) = -7.3827 \times 10^{-4}$$

$$f'(x_1) = -3.0433$$

$$\therefore x_2 = 0.518 - \frac{7.3287 \times 10^{-4}}{3.0433}$$

$$\therefore x_2 = 0.5177 \approx 0.518$$

$$\boxed{x_2 = 0.5177}$$

GAUSS ELEMINATION METHOD:-

→ i) $x + 4y - z = -5$

$x + y - 6z = -12$

$3x - y - z = 4$

ANS:

$$\begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & -6 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -12 \\ 4 \end{bmatrix}$$

Now, $R_2 \rightarrow R_2 - R_1$ $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 4 & -1 \\ 0 & -3 & -5 \\ 0 & -13 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -7 \\ 19 \end{bmatrix}$$

$R_3 \rightarrow 3R_3 - 13R_2$

$$\begin{bmatrix} 1 & 4 & -1 \\ 0 & -3 & -5 \\ 0 & 0 & 71 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -7 \\ 148 \end{bmatrix}$$

Now, $71z = 148$

$\boxed{z = 148/71 = \cancel{2.14}} \; 2.084$

$0x - 3y - 5z = -7$

$-3y - 5(2.084) = -7$

$3y + 105.7 = 7$

$y = \dfrac{7 - 105.7}{3} = -\dfrac{98.7}{3} = -1.14$

$\boxed{y = -1.14}$

$x + 4y - z = -5$

$x - 4.56 - 2.084 = -5$

$\boxed{x = 1.644}$

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→ **Gauss Elimination Method:-**

$10x - 7y + 3z + 5u = 6$ $\qquad u = 1$

$-6x + 8y - z - 4u = 5$ $\qquad z = -7$

$3x + y + 4z + 11u = 2$ $\qquad y = 4$

$5x - 9y - 2z + 4u = 7$ $\qquad x = 5$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ -6 & 8 & -1 & -4 \\ 3 & 1 & 4 & 11 \\ 5 & -9 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 2 \\ 7 \end{bmatrix}$$

$R_4 \to 2R_4 - R_1 \qquad R_3 \to 2R_3 + R_2$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ -6 & 8 & -1 & -4 \\ 0 & 10 & 7 & 18 \\ 0 & -25 & -7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 9 \\ 8 \end{bmatrix}$$

$R_2 \to 10R_2 + 6R_1$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 38 & 8 & -10 \\ 0 & 10 & 7 & 18 \\ 0 & -25 & -7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 86 \\ 9 \\ 8 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 38 & 8 & -10 \\ 0 & 10 & 7 & 18 \\ 0 & -15 & 0 & 21 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 86 \\ 9 \\ 187 \end{bmatrix}$$

$R_4 \to 2R_4 + 3R_3$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 38 & 8 & -10 \\ 0 & 10 & 7 & 18 \\ 0 & 0 & 21 & \cancel{138}\ 96 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 86 \\ 9 \\ 61 \end{bmatrix}$$

$\cancel{R_4 \to R_4 - 3R_3}$

$R_3 \to 38R_3 - 10R_2$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 38 & 8 & -10 \\ 0 & 0 & 186 & 784 \\ 0 & 0 & 21 & \cancel{138}\ 96 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 86 \\ -518 \\ 61 \end{bmatrix}$$

$R_4 \rightarrow 186R_4 - 21R_3$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 38 & 8 & -10 \\ 0 & 0 & 186 & 784 \\ 0 & 0 & 0 & \cancel{9204}\ 16092 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 86 \\ -518 \\ \cancel{61}\ 22224 \end{bmatrix}$$

Now, $u = \dfrac{22224}{\cancel{9204}\ 16092} = \cancel{2.414}\ \underline{1.38}$

$\boxed{u = 1.38}$

$186z + 784u = -518$

$z = \dfrac{-518 - 784(1.38)}{186} = -8.60$

$\boxed{z = -8.60}$

Also, $38y + 8z - 10u = 86.$

$\therefore 38y + 8(-8.60) - 10(1.38) = 86$

$\therefore 38y - 68.8 - 13.8 = 86$

$y = \dfrac{86 + 68.8 + 13.8}{38} = 4.43$

$\boxed{y = 4.43}$

Also, $10x - 7y + 3z + 5u = 6.$

$10x - 7(4.43) + 3(-8.60) + 5(1.38) = 6$

$10x - 31.01 - 25.8 + 6.9 = 6.$

$x = \dfrac{6 + 31.01 + 25.8 - 6.9}{10} = 5.591$

$\boxed{x = 5.6}$

Gauss Jordon Method:-

① $x + y + z = 9$

$2x + y - z = 0$

$2x + 5y + 7z = 52$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 52 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$; $R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -3 \\ 0 & 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -18 \\ 34 \end{bmatrix} \checkmark$$

$R_3 \rightarrow R_3 + 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -18 \\ -20 \end{bmatrix} \checkmark$$

$R_1 \rightarrow R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & -3 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \overset{-9}{\cancel{27}} \\ -18 \\ -20 \end{bmatrix} \checkmark$$

$R_1 \rightarrow 2R_1 \cancel{3} R_3$

$$\begin{bmatrix} 2 & \overset{1}{\cancel{3}} & \overset{-1}{\cancel{0}} \\ 0 & -1 & -3 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \overset{0}{\cancel{74}} \\ 18 \\ -20 \end{bmatrix}$$

~~$R_2 \rightarrow 4R_2 + 3R_3$~~

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 74 \\ 12 \\ -20 \end{bmatrix}$$

Solve using Gauss Elimination Method:-

→ $3x - y + 2z = 12$

$x + 2y + 3z = 11$

$2x - 2y - 1z = 2$

ANS. **SOLUTION:-** By Using Gauss Elimination Method.

$$\begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \\ 2 \end{bmatrix}$$

Now, $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & 2 \\ 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 12 \\ 2 \end{bmatrix}$$

$R_2 \Rightarrow R_2 - 3R_1$; $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -7 & -7 \\ 0 & -6 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -21 \\ -20 \end{bmatrix}$$

$R_2 \to R_2 / -7$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -6 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ \cancel{-21}\ 3 \\ -20 \end{bmatrix}$$

$R_3 \to R_3 + 6R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 3 \\ -2 \end{bmatrix}$$

Now, $x + 2y + 3z = 11$

$y + z = 3$

$-z = -2$ $\quad \therefore \boxed{z = 2}$

$y = 3 - 2 = 1$ $\quad \boxed{y = 1}$

And, $x + 2 + 6 = 11$

$x = 11 - 8 = 3$ $\quad \boxed{x = 3}$

NOTE:- UNIT 5 :- NUMERICAL METHODS.

1.) Soln of Algebraic or Transcendental Eqn
- → Bisection Method
- → Falsi position / Regula-Falsi
- → Seccant Method.
- → Newton Raphson Methon

2.) Soln of Simultaneous Equations.
- → Gauss Elimination Method.
- → Gauss Jordan Method.
- → LU Decomposition Method.
- → Jacobi iteration Method.
- → Gauss_Seidal Iterative Method.

⟹ (1.) Gauss_Elimination :- $\boxed{AX = B}$

A → Upper Triangular Form by using row transformations.

(2.) Gauss - Jordan : $AX = B$

A → Diagonal Matrix by using row transformation

$$A = \begin{bmatrix} x_1 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & z_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

(3.) LU Decomposition :- $AX = B$.

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$LUX = B$

Denote $UX = V$ ⟶ ①

→ $LV = B$ ⟶ we find the value of V.

Substitute in Eqn ①, we get value of x.

* LU Decomposition Method:-

→ $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

$$\boxed{AX = B}$$

$$\therefore \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ L_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$AX = B$$

$$LUX = B \longrightarrow ①$$

We will find the value of V, $LV = B$

Then, substitute in ② and get X.

Q) Apply LU Decomposition Method to solve the Equations

→ $3x + 2y + 7z = 4$

$2x + 3y + z = 5$

$3x + 4y + z = 7$

Ans. Soln:-

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + U_{32}U_{23} + U_{33} \end{bmatrix}$$

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Now, From R_1 : $\boxed{U_{11} = 3}$; $\boxed{U_{12} = 2}$; $\boxed{U_{13} = 7}$

Now, From R_2 :-

① $l_{21} U_{11} = 2$

$\therefore l_{21} \times 3 = 2$ $\boxed{l_{21} = 2/3 = \cancel{1.5}}$

② $l_{21} U_{12} + U_{22} = 3$

$\frac{2}{3} \times 2 + U_{22} = 3$

$U_{22} = 3 - \frac{4}{3} = \frac{9-4}{3} = \frac{5}{3}$ $\boxed{U_{22} = \frac{5}{3}}$

③ $l_{21} \cdot U_{13} + U_{23} = 1$

$\frac{2}{3} \times 7 + U_{23} = 1$

$U_{23} = 1 - \frac{14}{3} = \frac{3-14}{3} = \frac{-11}{3}$ $\boxed{U_{23} = -\frac{11}{3}}$

Now, From R_3 :-

① $l_{31} \cdot U_{11} = 3$

$l_{31} \times 3 = 3 \Rightarrow \boxed{l_{31} = 1}$

② $l_{31} \cdot U_{12} + l_{32} \cdot U_{22} = 4$

$1 \times 2 + l_{32} \times \frac{5}{3} = 4$

$l_{32} = \frac{(4-2) \times 3}{5} = \frac{2 \times 3}{5} = \frac{6}{5}$ $\boxed{l_{32} = 6/5}$

③ $l_{31} \cdot U_{13} + l_{32} \cdot U_{23} + U_{33} = 1.$

$1 \times 7 + \left(\frac{6}{5} \times -\frac{11}{3}\right) + U_{33} = 1$

$7 - \frac{66}{15} + U_{33} = 1 \Rightarrow U_{33} = 1 - 7 + \frac{66}{15} = \frac{-6+66}{15}$

$U_{33} = \frac{-90+66}{15} = \frac{-24}{15} = \frac{-8}{5}$ $\boxed{U_{33} = -\frac{8}{5}}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -9/3 \end{bmatrix}$$

$\boxed{AX = B}$ $\quad UX = V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \quad \therefore LX = B$

$LUX = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$V_1 = 4$

$2/3 V_1 + V_2 = 5 \qquad \therefore V_2 = \cancel{5-3/8} = 5 - 8/3 = 7/3$

$V_1 + 6/5 V_2 + V_3 = 7 \qquad \therefore V_3 = 1/5$

$UX = V.$

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7/3 \\ 1/5 \end{bmatrix}$$

$\therefore \frac{-8}{5} z = \frac{1}{5} \qquad \therefore \boxed{z = -1/8}$

$\therefore \frac{5}{3} y - \frac{11z}{3} = \frac{7}{3} \qquad \therefore \boxed{y = 9/8}$

$\therefore 3x + 2y + 7z = 4 \qquad \therefore \boxed{x = 7/8}$

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Ex.] solve by Gauss Elimination Method, the system of Equations :-

$\longrightarrow$

$2x_1 + x_2 + x_3 = 10$ $\qquad x = 7$

$3x_1 + 2x_2 + 3x_3 = 18.$ $\qquad y = -9$

$x_1 + 4x_2 + 9x_3 = 16$ $\qquad z = 5$

ANS. SOLUTION:-

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

$\therefore$ $R_1 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & 4 & 9 \\ 3 & 2 & 3 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 18 \\ 10 \end{bmatrix}$$

Now, $R_2 \rightarrow R_2 - 3R_1$; $R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & -10 & -24 \\ 0 & -7 & -17 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ -30 \\ -22 \end{bmatrix}$$

Now $R_2 \rightarrow \frac{-R_2}{2}$

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & 5 & 12 \\ 0 & -7 & -17 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 15 \\ -22 \end{bmatrix}$$

Now, $R_3 \rightarrow 5R_3 + 7R_2$

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & 5 & 12 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 15 \\ -5 \end{bmatrix}$$

Now, $-x_3 = -5$ $\qquad \boxed{x_3 = 5}$

$5x_2 + 12x_3 = 15$ $\qquad \boxed{x_2 = -9}$

$x_1 + 4x_2 + 9x_3 = 16$ $\qquad \boxed{x_3 = 5}$

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Ex.) Use LU Decomposition Method:-

→ $2x + y + 4z = 12$

$8x - 3y + 2z = 20$

$4x + 11y - z = 33$

ANS. SOLUTION:-

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21} U_{11} & l_{21}U_{12} + U_{22} & l_{21} U_{13} + U_{23} \\ l_{31} \cdot U_{11} & l_{31} U_{12} + l_{32} U_{22} & l_{31} U_{13} + l_{32} U_{23} + U_{33} \end{bmatrix}$$

Now, $\boxed{U_{11} = 2}$, $\boxed{U_{12} = 1}$, $\boxed{U_{13} = 4}$

$l_{21} \cdot U_{11} = 8 \Rightarrow l_{21} = 8/2 = 4$ $\boxed{l_{21} = 4}$

$l_{21} U_{12} + U_{22} = -3 \Rightarrow \boxed{U_{22} = -7}$

$l_{21} U_{13} + U_{23} = 2 \Rightarrow \boxed{U_{23} = -14}$

$l_{31} \cdot U_{11} = 4 \Rightarrow \boxed{l_{31} = 2}$

$l_{31} U_{12} + l_{32} U_{22} = 11 \Rightarrow \boxed{l_{32} = -9/7}$

$l_{31} \cdot U_{13} + l_{32} \cdot U_{23} + U_{33} = -1 \Rightarrow \boxed{U_{33} = -27}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -9/7 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -9/7 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 20 \\ 33 \end{pmatrix}$$

$\therefore \quad v_1 = 12$

$\therefore \quad 4v_1 + v_2 = 20 \implies \boxed{v_2 = -28}$

$\therefore \quad 2v_1 - \frac{9}{7} v_2 + v_3 = 33 \implies \boxed{v_3 = 4}$

$$\therefore \begin{pmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ -28 \\ 4 \end{pmatrix}$$

Now, $-27z = 4 \qquad \boxed{z = -4/27}$

$-7y - 14z = -28 \qquad -y - 2z = -4 \implies y + 2z = 4$

$\cancel{-7y + \frac{14 \times 4}{27} = -28} \implies \cancel{-7y = -28 - \frac{56}{27}} =$

$y = 4 - 2z = 4 + \frac{(2 \times 4)}{27} = \frac{108 + 8}{27} = \frac{116}{27}$

$\boxed{y = 116/27}$

$2x + y + 4z = 12$

$2x + \frac{116}{27} - \frac{16}{27} = 12 \implies 2x + \frac{116 - 16}{27} = 12$

$2x = 12 - \frac{100}{27} = \frac{324 - 100}{27} = \frac{224}{27}$

$x = \frac{224}{27 \times 2} = \frac{112}{27}$ $\cancel{x = 224}$

$\boxed{x = \frac{112}{27}}$

(4) JACOBI'S ITERATION METHOD:-

→
$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\} \longrightarrow (1)$$

If a_1, a_2, a_3 are large compared to the other coefficients, then solve x, y, z respectively

$$\left.\begin{aligned} x &= \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0) \\ y &= \frac{1}{b_2}(d_2 - a_2x_0 - c_2z_0) \\ z &= \frac{1}{c_3}(d_3 - a_3x_0 - b_3y_0) \end{aligned}\right\} \longrightarrow (2)$$

Consider intial approximations as x_0, y_0, z_0 for x, y, z.

∴ We substitute x_0, y_0, z_0 and get the first approximation

$$x_1 = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0)$$

$$y_1 = \frac{1}{b_2}(d_2 - a_2x_0 - c_2z_0)$$

$$z_1 = \frac{1}{c_3}(d_3 - a_3x_0 - b_3y_0)$$

Now, Substitute x_1, y_1, z_1 in (2), we get second Approximation.

$$x_2 = \frac{1}{a_1}(d_1 - b_1y_1 - c_1z_1)$$

$$y_2 = \frac{1}{b_2}(d_2 - a_2x_1 - c_2z_2)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3x_1 - b_3y_1)$$

This process continues till the difference between 2 (two) cosecutive approximations is negligible.

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Q.] **SOLVE:-** By Jacobi's Iteration Method, the Equations.

$$\left.\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \\ 2x - 3y + 20z &= 25 \end{aligned}\right\} \longrightarrow (1)$$

ANS. **Sol^n :-** Eqn (1) we write as:-

$$x = \frac{1}{20}(17 - y + 2z)$$

$$y = \frac{1}{20}(-18 - 3x + z)$$

$$z = \frac{1}{20}(25 - 2x + 3y)$$

Now, $x_0 = 0$, $y_0 = 0$, $z_0 = 0$ ... Consider this intial Approximation.

$\therefore$ 1st Approximation.

$$x_1 = \frac{1}{20}(17 - y_0 + 2z_0) = \frac{1}{20}(17 - 0 + 0) = \frac{17}{20} = 0.85$$

$$y_1 = \frac{1}{20}(-18 - 3x_0 + z_0) = \frac{1}{20}(-18 + 0 + 0) = \frac{-18}{20} = -0.9$$

$$z_1 = \frac{1}{20}(25 - 2x_0 + 3y_0) = \frac{1}{20}(25 - 0 + 0) = \frac{25}{20} = \frac{5}{4} = 1.25$$

2nd Approximation,

$$x_2 = \frac{1}{20}(17 - y_1 + 2z_1) = \frac{1}{20}(17 + 0.9 + 2.5) = 1.02.$$

$$y_2 = \frac{1}{20}(-18 - 3x_1 + z_1) = \frac{1}{20}(-18 - 2.55 + 1.25) = -0.965$$

$$z_2 = \frac{1}{20}(25 - 2x_1 + 3y_1) = \frac{1}{20}(25 - 1.7 - 2.7) = 1.03$$

3rd Approximation,

$$x_3 = \frac{1}{20}(17 - y_2 + 2z_2) = \frac{1}{20}(17 + 0.965 + 2.06) = 1.00125$$

$$y_3 = \frac{1}{20}(-18 - 3x_2 + z_2) = \frac{1}{20}(-18 - 3.06 + 1.03) = -1.0015$$

$$z_3 = \frac{1}{20}(25 - 2x_2 + 3y_2) = \frac{1}{20}(25 - 2.04 - 2.895) = 1.00325$$

4th Approximation:-

$$x_4 = \frac{1}{20}(17 - y_3 + 2z_3) = \frac{1}{20}(17 + 1.0015 + 2.0065) = 1.0004$$

$$y_4 = \frac{1}{20}(-18 - 3x_3 + z_3) = \frac{1}{20}(-18 - 3.00375 + 1.00325) = -1.000078$$

$$z_4 = \frac{1}{20}(25 - 2x_3 + 3y_3) = \frac{1}{20}(25 - 2.0025 - 3.0045) = 0.99965$$

5th Approximation:-

$$x_5 = \frac{1}{20}(17 - y_4 + 2z_4) = 0.999966$$

$$y_6 = \frac{1}{20}(-18 - 3x_4 + z_4) = -1.000078$$

$$z_5 = \frac{1}{20}(25 - 2x_4 + 3y_4) = 0.999956$$

6th Approximation:-

$$x_6 = \frac{1}{20}(17 - y_5 + 2z_5) = 1.0000$$

$$y_6 = \frac{1}{20}(-18 - 3x_5 + z_5) = -0.999997$$

$$z_6 = \frac{1}{20}(25 - 2x_5 + 3y_5) = 0.999992$$

$x \cong 1, \; y \cong 1, \; z \cong 1$

Q.) Solve:- by Jacobi's iteration Method, the Equations.

→ $10x + y - z = 11.19$

$x + 10y + z = 28.08$

$-x + y + 10z = 35.61$

Correct to two decimal places.

→ $x = \frac{1}{10}(11.19 - y + z)$

$y = \frac{1}{10}(28.08 - x - z)$

$z = \frac{1}{10}(35.61 + x - y)$

Now, $x_0 = 0, y_0 = 0, z_0 = 0$.

$\therefore x_1 = \frac{1}{10}(11.19 - y_0 + z_0) = \frac{1}{10}(11.19) = \frac{11.19}{10} = 1.119 = 1.11$

$\therefore y_1 = \frac{1}{10}(28.08 - x_0 - z_0) = \frac{1}{10}(28.08) = \frac{28.08}{10} = 2.80$

$\therefore z_1 = \frac{1}{10}(35.61 - x_0 - y_0) = \frac{1}{10}(35.61) = \frac{35.61}{10} = 3.56$

2nd iteration:-

$x_2 = \frac{1}{10}(11.19 - y_1 + z_1) = \frac{1}{10}(11.19 - 2.80 + 3.56) = 1.19$

$y_2 = \frac{1}{10}(28.08 - x_1 - z_1) = \frac{1}{10}(28.08 - 1.11 - 3.56) = 2.34$

$z_2 = \frac{1}{10}(35.61 - x_1 - y_1) = \frac{1}{10}(35.61 - 1.11 - 2.80) = 3.39$.

3rd iteration:-

$x_3 = \frac{1}{10}(11.19 - y_2 + z_2) = 1.22$

$y_3 = \frac{1}{10}(28.08 - x_2 - z_2) = 2.35$

$z_3 = \frac{1}{10}(35.61 - x_2 - y_2) = 3.45$

4th Iteration :-

$$x_4 = \frac{1}{10}(11.19 - y_3 + z_3) = 1.23$$

$$y_4 = \frac{1}{10}(28.08 - x_3 - z_3) = 2.34$$

$$z_4 = \frac{1}{10}(35.61 - x_3 - y_3) = 3.45$$

5th iteration :-

$x_5 = 1.23$

$y_5 = 2.34$

$z_5 = 3.45$

Hence, $x = 1.23$, $y = 2.34$ and $z = 3.45$

(B) **Gauss - seidal Method :-**

→ $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z + d_3$

Now, $x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$

$y = \frac{1}{b_2}(d_2 - b_2x - c_2z)$

$z = \frac{1}{c_2}(d_3 - a_3x - b_3y$

x_0, y_0, z_0 $\quad x_1 = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0)$

$y_1 = \frac{1}{b_2}(d_2 - a_2x_1 - c_2z_0)$

$z_1 = \frac{1}{c_3}(d_3 - a_3x_1 - b_3y_1)$

2^{nd} Iteration,

$$x_2 = \frac{1}{a_1}(d_1 - b_1 y_1 - c_1 z_1)$$

$$y_2 = \frac{1}{b_2}(d_2 - a_2 x_2 - c_2 z_1)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3 x_2 - b_3 y_2)$$

& so on.

Ex. $20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

ANS. soln :-

$$x = \frac{1}{20}(17 - y + 2z)$$

$$y = \frac{1}{20}(-18 - 3x + z)$$

$$z = \frac{1}{20}(25 - 2x + 3y)$$

Now, $x_0 = 0$, $y_0 = 0$, $z_0 = 0$.

1^{st} Approximation:-

$$x_1 = \frac{1}{20}(17 - y_0 + 2z_0) = \frac{17}{20} = 0.8500$$

$$y_1 = \frac{1}{20}(-18 - 3x_1 + z_0) = \frac{(-18 - 2.55)}{20} = -1.0275$$

$$z_1 = \frac{1}{20}(25 - 2x_1 + 3y_1) = \cancel{\frac{25}{20}} = \cancel{\frac{5}{4}} = \frac{1}{20}(25 - 1.7 - 3.0825)$$

$\therefore z_1 = 1.010875 \approx 1.0109$

2^{nd} Approximation:-

$$x_2 = \frac{1}{20}(17 - y_1 + 2z_1) = 1.0025$$

$$y_2 = \frac{1}{20}(-18 - 3x_2 + z_1) = -0.9998$$

$$z_2 = \frac{1}{20}(25 - 2x_2 + 3y_2) = 0.9998$$

3^{rd} Approximation:-

$$x_3 = \frac{1}{20}(17 - y_2 + 2z_2) = 1.0000$$

$$y_3 = \frac{1}{20}(-18 - 3x_2 - z_2) = -1.0000$$

$$z_3 = \frac{1}{20}(25 - 2x_3 + 3y_3) = 1.0000$$

Ex.

$$4x + 2y + z = 14$$
$$x + 5y - z = 10$$
$$x + y + 8z = 20.$$

ANS. Soln :-

$$x = \frac{1}{4}(14 - 2y - z), \quad y = \frac{1}{5}(10 + z - x); \quad z = \frac{1}{8}(20 - x - y)$$

$x_0 = 0, \quad y_0 = 0, \quad z_0 = 0.$

1^{st} iteration:-

$$x_1 = \frac{1}{4}(14 - 2y_0 - z_0) = \frac{1}{4} \times 14 = 3.5$$

$$y_1 = \frac{1}{5}(10 - x_1 + z_0) = \frac{1}{5}(10 - 3.5 + 0) = \frac{6.5}{5} = 1.3$$

$$z_1 = \frac{1}{8}(20 - x_1 - y_1) = \frac{1}{8}(20 - 3.5 - 1.3) = 1.9$$

2nd Iteration:-

$$x_2 = \frac{1}{4}(14 - 2y_1 - z_1) = \frac{1}{4}(14 - 2.6 - 1.9) = \underline{\underline{2.375}}$$

$$y_2 = \frac{1}{5}(10 - x_2 + z_1) = \frac{1}{5}(10 - 2.375 + 1.9) = \underline{\underline{1.905}}$$

$$z_2 = \frac{1}{8}(20 - x_2 - y_2) = \frac{1}{8}(20 - 2.375 - 1.905) = \underline{\underline{1.965}}$$

3rd Iteration:-

$$x_3 = \frac{1}{4}(14 - 2y_2 - z_2) = 2.07125$$

$$y_3 = \frac{1}{5}(10 - x_3 + z_2) = 1.98$$

$$z_3 = \frac{1}{8}(20 - x_3 - y_3) = 1.99$$

4th Iteration:-

$$x_4 = \frac{1}{4}(14 - 2y_3 - z_3) = 2.0125$$

$$y_4 = \frac{1}{5}(10 - x_4 + z_3) = 1.995$$

$$z_4 = \frac{1}{8}(20 - x_4 - y_4) = 1.999$$

Hence, $x = 2.07125 \approx 2$

$y = 1.98 \approx 2$

$z = 1.99 \approx 2$