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FRIDAY.

• TUTORIAL - 1 (ONE):-

EXERCISE : 1.1 :-

(1.) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0.$

ANS. Given: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0 \Rightarrow \left(\frac{d^2}{dx^2} - 5\frac{d}{dx} - 6\right)y = 0$

Let, $D = \frac{d}{dx}$

$\therefore (D^2 - 5D - 6).y = 0 \longrightarrow (1)$

The Auxiliary Equation (A.E.) for (1) is $D^2 - 5D - 6 = 0.$

$\therefore D^2 - 5D - 6 = 0$

$\therefore D^2 - 6D + 1D - 6 = 0$

$\therefore D(D-6) + 1(D-6) = 0$

$\therefore (D+1)(D-6) = 0$

$\therefore D + 1 = 0 \quad D - 6 = 0$

$\boxed{D = -1}\ (m_1) \quad \boxed{D = 6}\ (m_2)$

$\therefore$ The roots of Equation are real and distinct.

Complimentary Function (CF): $c_1e^{m_1x} + c_2e^{m_1x}$

The General solution of the Equation is :-

$y = c_1e^{-x} + c_2e^{6x}$

$\boxed{y = c_1e^{-x} + c_2 \cdot e^{6x}}$

(2.) $2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 10y = 0$

ANS. Given: $2 \cdot \frac{d^2y}{dx^2} - \frac{dy}{dx} - 10y = 0 \Rightarrow \left(2 \cdot \frac{d^2}{dy^2} - \frac{d}{dx} - 10\right)y = 0$

Let, $D = \frac{d}{dx}$

$\therefore (2D^2 - D - 10)y = 0. \longrightarrow (1)$

The Auxiliary Equation (A.E.) for (1) is $2D^2 - D - 10 = 0$

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$\therefore\ 2D^2 - D - 10 = 0$

$-5 \quad +4$

$\therefore\ 2D^2 - 5D + 4D - 10 = 0$

$\therefore\ 2D(D - 5/2) + 4(D - 5/2) = 0$

$\therefore\ \left(D - \frac{5}{2}\right)(2D + 4) = 0$

$\therefore\ D - \frac{5}{2} = 0 \qquad 2D + 4 = 0$

$\qquad\qquad\qquad 2D = -4$

$\therefore\ \boxed{D = 5/2}\ (m_1) \qquad \boxed{D = -2}\ (m_2)$

The roots of Equation are Real and Distinct.

Complimentary Function (C.F.) : $c_1 e^{m_1 x} + c_2 e^{m_1 x}$

The General Solution of the Differential Equation is :-

$y = c_1 \cdot e^{5/2x} + c_2 e^{-2x}$

$$\boxed{y = c_1 \cdot e^{5/2x} + c_2 \cdot e^{-2x}}$$

(3.) $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Ans. Given:- $\frac{d^3y}{dx^3} + 2\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0 \Rightarrow \left(\frac{d^3}{dx^3} + 2\frac{d^2}{dy^2}\right) y + \frac{d}{dx} D = 0$

Let , $D = \frac{d}{dx}$

$\therefore (D^3 + 2D^2 + D)\cdot y = 0 \longrightarrow (1)$

The Auxiliary Equation (A.E.) for (1) is $D^3 + 2D^2 + D = 0$

$\therefore\ D^3 + 2D^2 + D = 0.$

The roots of the equation are

$2D^2 = D^2 + D^2$

$\therefore\ D^3 + D^2 + D^2 + D = 0.$

$\therefore\ D^2(D + 1) + D(D + 1) = 0$

$\therefore\ (D^2 + D)(D + 1) = 0$

$\therefore\ D^2 + D = 0 \ ; \quad D + 1 = 0$

$D(D + 1) = 0 \ ; \quad D = -1$

$D = 0 \ ; \ D + 1 = 0 \ ; \quad D = -1$

$\therefore\ \boxed{D = 0, -1, -1}$

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① for $D = 0$

if $D = a$

$\boxed{(C.F.)_1 = c_1 e^{ax}} \longrightarrow (1)$

② For $D = -1, -1$;

if $D = a, a$

$\boxed{(C.F.)_2 = e^{ax}(c_2 + c_3 x)} \longrightarrow (2)$

The General solution of the Equation is:-

$y = C.F. = (C.F.)_1 + (C.F.)_2 = c_1 e^{ax} + e^{ax}(c_2 + c_3 x)$

$\therefore \boxed{y = c_1 + (c_2 + c_3 x)e^{-x}}$

(4.) $(D^4 - 2D^3 + D^2)y = 0.$

ANS. Given : $(D^4 - 2D^3 + D^2)y = 0. \longrightarrow (1)$

$D^4 - 2D^3 + D^2 = 0$ is an Auxiliary Equation (A.E.) of (1)

$\therefore D^4 - D^3 - D^3 + D^2 = 0$

$\therefore D^3(D-1) - D^2(D-1) = 0$

$\therefore (D^3 - D^2)(D-1) = 0$

$\therefore D^3 - D^2 = 0$; $D - 1 = 0.$

$D^2(D-1) = 0$; $\boxed{D = 1}$

$D^2 = 0$; $D - 1 = 0$

$\boxed{D = 0}$ $\boxed{D = 1}$ $\boxed{D = 0, 1, 1, 0}$

① For $D = 0, 0$

if $D = a, a$

then $(C.F.)_1 = (c_1 + c_2 x) \cdot e^{ax}$

② For $D = 1, 1.$

if $D = a, a$

then $(C.F.)_2 = (c_3 + c_4 x)e^{ax}$

The General Solution of the Equation is:-

$y = C.F. = (C.F.)_1 + (C.F.)_2$

$= (c_1 + c_2 x) \cdot e^{0 \cdot x} + (c_3 + c_4 x) \cdot e^{x}$

$\boxed{y = c_1 + c_2 x + e^{x}(c_3 + c_4 \cdot x)}$

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(5.) $(D^3 + 6D^2 + 11D + 6)y = 0$

ANS. Given :- $(D^3 + 6D^2 + 11D + 6)y = 0 \longrightarrow (1)$

The Auxiliary Equation (A.E.) of (1) is $D^3 + 6D^2 + 11D + 6 = 0$

Let, $p(x) \Rightarrow x^3 + 6x^2 + 11x + 6 = 0$

Put $x = -1$

$\therefore p(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6.$

$\therefore p(-1) = -1 + 6(1) + (-11) + 6 = -1 + 6 - 11 + 6$

$\therefore p(-1) = -12 + 12 = \underline{0}.$

$\therefore x = -1$ i.e. $x + 1 = 0$

$(x+1)$ is a factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2 + 5x + 6 \\
\hline
x+1 & \cancel{x^3} + 6x^2 + 11x + 6 \\
- & \cancel{x^3} + x^2 \\
\hline
 & \cancel{5x^2} + 11x + 6 \\
- & \cancel{5x^2} + 5x \\
\hline
 & \cancel{6x} + \cancel{6} \\
- & \cancel{6x} + \cancel{6} \\
\hline
 & 0 \quad 0
\end{array}
$$

$\therefore Q = x^2 + 5x + 6$ .... (Quadratic Equation)

$Q = x^2 + 3x + 2x + 6$

$Q = x(x+3) + 2(x+3)$ ~~= 0~~

$Q = (x+3)(x+2)$

$\therefore p(x) = (x+1)(x+2)(x+3)$

Thus, The roots of the Differential Equation are:-

$\boxed{x = -1, -2, -3}$

- The General Solution of the Equation is:-

$\boxed{y = c_1 \cdot e^{-x} + c_2 \cdot e^{-2x} + c_3 \cdot e^{-3x}}$ ✓

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(6) $4y'' - 8y' + 7y = 0$

ANS. Given:- $4y'' - 8y' + 7y = 0$

Let, $D = \frac{d}{dx} = \frac{y'}{y}$

$\therefore 4D^2y - 8Dy + 7y = 0.$

$\therefore (4D^2 - 8D + 7)y = 0 \longrightarrow (1)$

The Auxiliary Equation (A.E.) of (1) is $4D^2 - 8D + 7 = 0$

$\therefore D = \alpha \pm \beta i = 1 + \frac{\sqrt{3}}{2} i$

Now, comparing $D = 1 + \sqrt{3}/2\, i$ with $D = \alpha + \beta i$; we get,

$\alpha = 1$ and $\beta = \sqrt{3}/2$

$C.F. = e^{\alpha x}[c_1 \cos \beta x + c_2 \sin \beta x]$

The General Solution for the above Differential Equation is:-

$$\boxed{y = C.F. = e^{x}\left[c_1 \cdot \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 \cdot \sin\left(\frac{\sqrt{3}}{2}x\right)\right]}$$

(7) $(D^3 + D^2 - 2D + 12)y = 0$

ANS. Given:- $(D^3 + D^2 - 2D + 12)y = 0. \longrightarrow (1)$

$\therefore$ The Auxiliary Equation (A.E.) of (1) is

$D^3 + D^2 - 2D + 12 = 0$

The roots of the above Equation is:-

$$\boxed{D = -3,\ 1 + \sqrt{3}i,\ 1 - \sqrt{3}i}$$

① For $D = -3$

if $D = a$

then $(C.F.)_1 = c_1 . e^{ax}$

② For $D = 1 \pm \sqrt{3}\, i$

if $D = \alpha \pm \beta i$

then $(C.F.)_2 = e^{\alpha x}[c_2 \cdot \cos(\sqrt{3}x) + c_3 \cdot \sin(\sqrt{3}x)]$

The General Solution of the D.E. is:-

$y = C.F. = (C.F.)_1 + (C.F.)_2$

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$$y = c_1 \cdot e^{-3x} + e^{x}[c_2 \cdot \cos(\sqrt{3}x) + c_3 \cdot \sin(\sqrt{3}x)]$$

$$\boxed{y = c_1 \cdot e^{-3x} + e^{x}[c_2 \cdot \cos(\sqrt{3}x) + c_3 \cdot \sin(\sqrt{3}x)]}$$

(8.) $4 \cdot \frac{d^2s}{dt^2} = -9s$

ANS. Given: $4 \cdot \frac{d^2s}{dt^2} = -9s$

Let, $D = \frac{d}{dt}$ $D^2 = \frac{d^2}{dt^2}$

$\therefore 4D^2 s + 9s = 0$

$\therefore (4D^2 + 9)s = 0 \longrightarrow (1)$

The Auxiliary Equation (A.E.) of (1) is $4D^2 + 9 = 0$

$D^2 = \frac{-9}{4} \Rightarrow D = \pm\sqrt{\frac{-9}{4}} = \pm\frac{3}{2}$

$\boxed{D = \pm\frac{3}{2}}$ $\therefore D = \underset{(\alpha)}{0} \pm \underset{(\beta)}{\frac{3}{2}}i$

if $D = \alpha \pm \beta i$

then $CF = e^{\alpha x}[c_1 \cos\beta x + c_2 \cdot \sin\beta x]$

The General solution of the Differential Equation is:-

$y = CF = e^{\alpha x}[c_1 \cdot \cos(\beta x) + c_2 \cdot \sin(\beta x)]$

$\therefore y = e^{(0)x}[c_1 \cdot \cos(\frac{3}{2}x) + c_2 \cdot \sin(\frac{3}{2}x)]$

A.) $\boxed{y = c_1 \cdot \cos\left(\frac{3}{2}x\right) + c_2 \cdot \sin\left(\frac{3}{2}x\right)}$

(9.) $(D^6 - 6D^5 + 12D^4 - 6D^3 - 9D^2 + 12D - 4)y = 0$

ANS. Given: $(D^6 - 6D^5 + 12D^4 - 6D^3 - 9D^2 + 12D - 4)y = 0 \longrightarrow (1)$

The Auxiliary Equation (A.E.) of above Equation is:-

$D^6 - 6D^5 + 12D^4 - 6D^3 - 9D^2 + 12D - 4 = 0$

The roots of the above equation is:-

$\boxed{D = 1, 1, 1, 2, 2, -1}$

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① For $D = 1, 1, 1$

if $D = a, a, a$.

then $(CF)_1 = [c_1 + c_2 x + c_3 x^2] \cdot e^{ax}$ ——— (i)

② For $D = 2, 2$.

if $D = a, a$.

then $(CF)_2 = [c_4 + c_5 \cdot x] \cdot e^{ax}$ ——— (ii)

③ For $D = -1$

if $D = a$

then $(CF)_3 = c_6 \cdot e^{ax}$ ——— (iii)

The General Solution for the D.E. is:-

$y = CF = (CF)_1 + (CF)_2 + (CF)_3$

$\therefore y = [c_1 + c_2 x + c_3 x^2] \cdot e^{ax} + [c_4 + c_5 x] e^{ax} + c_6 \cdot e^{ax}$

$$\Rightarrow \boxed{y = (c_1 + c_2 x + c_3 \cdot x^2) \cdot e^{x} + (c_4 + c_5 x) e^{2x} + c_6 \cdot e^{-1x}}$$

(10) $\frac{d^2y}{dx^2} - 9y = 0$

ANS. Given: $\frac{d^2y}{dx^2} - 9y = 0 \longrightarrow (1)$

Let, $D = \frac{d}{dx} \Rightarrow D^2 = \frac{d^2}{dx^2}$

$\therefore (D^2 - 9) y = 0$.

The Auxiliary Equation (A.E.) of above is $D^2 - 9 = 0$

$D^2 - 9 = 0$.

$\therefore D^2 - (3)^2 = 0$

$\therefore (D-3)(D+3) = 0$.

$\therefore \boxed{D = 3}$; $\boxed{D = -3}$

The roots are real and distinct.

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if $D = a_1, a_2$

then $CF = c_1 \cdot e^{a_1 x} + c_2 \cdot e^{a_2 x}$

Thus, The General solution of the Equation is:-

$y = C.F.$

$$\boxed{y = c_1 \cdot e^{-3x} + c_2 \cdot e^{3x}}$$