Unit II

Hypothesis testing:
Hypothesis testing is a statistical method used to make inferences or decisions about a population based on sample data. It involves testing an assumption (hypothesis) about a population parameter.

Steps in Hypothesis Testing

1. **State the Null and Alternative Hypotheses**
   - **Null Hypothesis (H_0):** Assumes no effect or no difference (e.g., "The average height of students is 5.5 feet").
   - **Alternative Hypothesis (H_1 or Ha):** Represents the claim being tested (e.g., "The average height of students is not 5.5 feet").
2. **Choose a Significance Level (α)**
   - Common values: 0.05 (5%), 0.01 (1%), 0.10 (10%)
   - This is the probability of rejecting the null hypothesis when it is actually true (Type I error).
3. **Select a Test Statistic and Determine the Sampling Distribution**
   - Choose a statistical test based on data type and sample size (e.g., Z-test, t-test, chi-square test, ANOVA).
   - Compute the test statistic (e.g., z-score, t-score).
4. **Calculate the p-value or Critical Value**
   - **p-value:** The probability of obtaining a test result as extreme as the observed one, assuming H_0 is true.
   - **Critical value approach:** Compare the test statistic to a predefined critical value.
5. **Make a Decision**
   - If **p-value ≤ α**, reject H_0 (significant result).
   - If **p-value > α**, fail to reject H_0 (not enough evidence to support H_1).
6. **Interpret the Results**
   - Explain in the context of the problem whether there is enough evidence to support the alternative hypothesis.

Types of Hypothesis Tests

1. **Z-test** (for large sample sizes, n > 30)
2. **t-test** (for small samples, n < 30)
   - One-sample t-test
   - Two-sample t-test
   - Paired t-test
3. **Chi-square test** (for categorical data)
4. **ANOVA** (for comparing means across multiple groups)
5. **Regression analysis** (to test relationships between variables)]

Let's go through an example of a **one-sample t-test**, which is used when we want to compare the mean of a sample to a known population mean when the population standard deviation is unknown.

Example: (One-T test)Testing the Average Sleep Hours of Students

A researcher believes that college students sleep **less than 7 hours per night on average**. To test this claim, they collect a random sample of **15 students** and record their sleep hours. The sample mean is **6.5 hours**, and the sample standard deviation is **1.2 hours**. The researcher sets a significance level of **0.05**.

Step 1: State the Hypotheses

- **Null Hypothesis (H_0):** The average sleep duration of students is 7 hours.
  $H0:\mu=7$
- **Alternative Hypothesis (H_1):** Students sleep less than 7 hours on average.
  $H1:\mu<7$ (This is a **left-tailed test**)

Step 2: Choose the Significance Level

$\alpha=0.05$

Step 3: Compute the Test Statistic (t-score)

The **t-score** is calculated using the formula:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Where:

- $\bar{x} = 6.5$ (sample mean)
- $\mu = 7$ (population mean)
- $s = 1.2$ (sample standard deviation)
- $n = 15$ (sample size)

Plugging in the values:

$$t = \frac{6.5 - 7}{1.2/\sqrt{15}}$$

Let's calculate this value.

The calculated t-score is **-1.61**.

Step 4: Find the Critical Value (t-critical)

For a **one-tailed t-test** at $\alpha=0.05$ with **degrees of freedom (df) = n - 1 = 15 - 1 = 14**, we look up the t-table or use statistical software to find the critical t-value.

The critical value for $t_{0.05,14}$ (left-tailed) is approximately **-1.761**.

Step 5: Make a Decision

- Our **calculated t-score** is **-1.61**.
- The **critical t-value** is **-1.761**.
- Since t>tcritical (**-1.61 > -1.761**), we **fail to reject the null hypothesis**.

Step 6: Conclusion

There is **not enough evidence** to conclude that students sleep less than 7 hours per night on average at the 5% significance level.

Let's go through an example using a **two-sample t-test**, which compares the means of two independent groups.

Example: (Two Test)Testing the Effectiveness of a New Diet Plan

A nutritionist wants to test whether a **new diet plan** leads to **more weight loss** compared to a standard diet. Two groups of people follow different diets for **6 weeks**:

- **Group A (New Diet)**: 12 participants, average weight loss = **6.8 kg**, standard deviation = **2.1 kg**
- **Group B (Standard Diet)**: 10 participants, average weight loss = **5.2 kg**, standard deviation = **1.8 kg**

The researcher sets a significance level of **0.05**.

Step 1: State the Hypotheses

- **Null Hypothesis (H_0):** There is no difference in weight loss between the two diets.
  $H0:\mu A=\mu B$
- **Alternative Hypothesis (H_1):** The new diet leads to **greater weight loss** than the standard diet.
  $H1:\mu A>\mu B$ (This is a **right-tailed test**)

Step 2: Choose the Significance Level

$\alpha=0.05$

Step 3: Compute the Test Statistic (t-score)

The formula for a **two-sample t-test** (assuming unequal variances) is:

The

$$t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

Where:

- $\bar{x}_A = 6.8$, $s_A = 2.1$, $n_A = 12$ (New Diet group)
- $\bar{x}_B = 5.2$, $s_B = 1.8$, $n_B = 10$ (Standard Diet group)

Let's calculate the **t-score**.

calculated **t-score** is **1.92**.

Step 4: Find the Critical Value (t-critical)

For a **one-tailed t-test** at $\alpha = 0.05$, we need the **degrees of freedom (df)**, which is estimated using:

$$df = \frac{\left(\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}\right)^2}{\frac{\left(\frac{s_A^2}{n_A}\right)^2}{n_A - 1} + \frac{\left(\frac{s_B^2}{n_B}\right)^2}{n_B - 1}}$$

Let's calculate the **degrees of freedom (df)** and find the **critical t-value**. [>-]

The **degrees of freedom (df)** is approximately **19.97**, and the **critical t-value** for a one-tailed test at α=0.05 is **1.72**.

Step 5: Make a Decision

- Our **calculated t-score** is **1.92**.
- The **critical t-value** is **1.72**.
- Since t>tcritical (**1.92 > 1.72**), we **reject the null hypothesis**.

Step 6: Conclusion

At the **5% significance level**, there is **sufficient evidence** to conclude that the **new diet plan leads to greater weight loss** compared to the standard diet.

Need for Hypothesis Testing

Hypothesis testing is essential for making **scientific and data-driven decisions** in various fields. Here's why:

1. **Decision-Making in Uncertainty**
    - Helps in making informed conclusions based on sample data rather than guessing.
2. **Validates Assumptions**
    - Ensures claims or theories are statistically backed (e.g., effectiveness of a new drug).
3. **Avoids Bias and Subjectivity**
    - Provides an objective way to analyze data and avoid misleading conclusions.
4. **Used in Business, Medicine, and Research**
    - Businesses use it to test marketing strategies.
    - Doctors use it to check treatment effectiveness.
    - Scientists use it to validate experimental results.

Pearson Correlation Coefficient (r)

The **Pearson correlation coefficient** measures the **strength and direction** of the **linear relationship** between two continuous variables. It is denoted by **r** and ranges from **-1 to 1**.

Formula for Pearson's r

Formula for Pearson's r

$$r = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum(X_i - \bar{X})^2} \times \sqrt{\sum(Y_i - \bar{Y})^2}}$$

Where:

- X_i and Y_i are individual data points,
- $\bar{X}$ and $\bar{Y}$ are the means of X and Y,
- $\sum$ represents summation.

Where:

- Xi and Yi are individual data points,
- X⁻ and Y⁻ are the means of X and Y,
- ∑ represents summation.

Interpretation of Pearson's r

Value of r	Interpretation
r=1	Perfect **positive** correlation (as X increases, Y increases).

Value of r	Interpretation
$r>0$	Positive correlation (X and Y move in the same direction).
$r=0$	No correlation (X and Y are unrelated).
$r<0$	Negative correlation (X and Y move in opposite directions).
$r=-1$	Perfect **negative** correlation (as X increases, Y decreases).

Example: Pearson Correlation in Action

Let's say we have **student study hours (X)** and their **exam scores (Y)**:

Study Hours (X)	Exam Score (Y)
2	50
3	60
5	80
7	90
8	95

The **Pearson correlation coefficient (r)** is **0.987**, indicating a **strong positive correlation** between study hours and exam scores. This means that as study hours increase, exam scores tend to increase as well.

Sample Hypothesis testing:-

One-Sample Hypothesis Test: Explanation & Example

A **one-sample hypothesis test** is used to determine whether the mean of a single sample is significantly different from a known **population mean**.

Example: Testing Average Daily Screen Time

A researcher claims that the average daily screen time for teenagers is **6 hours**. To test this claim, a sample of **15 teenagers** is surveyed, and their screen times (in hours) are recorded:

Sample Data (daily screen time in hours):

5.5, 6.2, 7.1, 5.8, 6.5, 7.0, 5.9, 6.1, 6.8, 7.2, 6.3, 6.0, 6.7, 5.6, 6.4

The researcher sets a significance level of **0.05**.

Step 1: State the Hypotheses

- **Null Hypothesis (H0)**: The average daily screen time is **6 hours**. H0:μ=6
- **Alternative Hypothesis (H1)**: The average daily screen time is **not equal to 6 hours**. H1 :μ�=6 *(This is a* **two-tailed** *test because we are testing for any difference, not just greater or smaller.)*

Step 2: Choose the Significance Level

- α=0.05 (5%)

Step 3: Compute the Test Statistic (t-score)

Where:

The formula for a **one-sample t-test** is:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Where:

- $\bar{x}$ = sample mean
- μ = population mean (6 hours)
- s = sample standard deviation
- n = sample size (15)

- $\bar{x}$ = sample mean
- μ = population mean (6 hours)
- s = sample standard deviation
- n = sample size (15)

Let's calculate the **t-score**.

The **calculated t-score** is **2.45**.

Step 4: Find the Critical Value (t-critical)

For a **two-tailed t-test** at α=0.05 with **degrees of freedom (df) = n - 1 = 15 - 1 = 14**, we find the **critical t-value**. Let's calculate it.

The **critical t-value** for a two-tailed test at α=0.05 with 14 degrees of freedom is **2.14**.

Step 5: Make a Decision

- Our **calculated t-score** is **2.45**.
- The **critical t-value** is **2.14**.
- Since $|t| > t_{critical}$ (**2.45 > 2.14**), we **reject the null hypothesis**.

Step 6: Conclusion

At the **5% significance level**, there is **sufficient evidence** to conclude that the **average daily screen time for teenagers is significantly different from 6 hours**.

Two-Sample Hypothesis Test: Explanation & Example:

A **two-sample hypothesis test** is used to compare the means of two independent groups to determine if there is a significant difference between them.

Example: Comparing Study Hours of Male and Female Students

A school principal wants to determine whether **male and female students study for the same number of hours per week** on average. To test this, a random sample of students is taken:

Sample Data (Study Hours per Week):

- **Male Students: 12, 15, 14, 10, 13, 16, 11, 14, 13, 12**
- **Female Students: 18, 16, 17, 15, 19, 20, 14, 16, 15, 17**

The principal sets a significance level of **0.05**.

Step 1: State the Hypotheses

- **Null Hypothesis (H0)**: The average study hours for male and female students are equal. H0 :μ1=μ2
- **Alternative Hypothesis (H1)**: The average study hours for male and female students are **not equal**. H1:μ1❓=μ2 *(This is a* **two-tailed** *test because we are testing for any difference, not just greater or smaller.)*

Step 2: Choose the Significance Level

- α=0.05 (5%)

Step 3: Compute the Test Statistic (t-score)

The formula for a **two-sample t-test** (assuming unequal variances) is:

Let's

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Where:

- $\bar{x}_1, \bar{x}_2$ = sample means of groups
- s_1, s_2 = sample standard deviations of groups
- n_1, n_2 = sample sizes

calculate the **t-score**.

The **calculated t-score** is **-4.45**, indicating a significant difference between the two groups.

Step 4: Find the Critical Value (t-critical)

For a **two-tailed t-test** at $\alpha = 0.05$, we determine the degrees of freedom (df) using the **Welch-Satterthwaite equation**:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$$

Let's calculate the **degrees of freedom** and find the **critical t-value.** [>-]

The **degrees of freedom (df)** is approximately **17.98**, and the **critical t-value** for a two-tailed test at α=0.05 is **2.10**.

Step 5: Make a Decision

- Our **calculated t-score** is **-4.45**.
- The **critical t-value** is **±2.10**.
- Since ∣t∣>tcritical (**4.45 > 2.10**), we **reject the null hypothesis**.

Step 6: Conclusion

At the **5% significance level**, there is **strong evidence** to conclude that the **average study hours for male and female students are significantly different**.

t-Test: Explanation & Types:

A **t-test** is a statistical test used to compare the means of one or two groups to determine if there is a significant difference between them. It is used when the **population standard deviation is unknown** and the sample size is small (n<30).

Types of t-Tests

1. **One-Sample t-Test**
    - Compares the mean of a single sample to a known population mean.
    - Example: "Is the average height of students **equal to 170 cm**?"
2. **Two-Sample t-Test (Independent t-Test)**
    - Compares the means of **two independent groups**.
    - Example: "Do **male and female students** have different study hours per week?"

3. **Paired t-Test (Dependent t-Test)**
    - Compares the means of **two related (paired) samples**, such as **before-and-after** measurements.
    - Example: "Does a **training program** improve test scores **before and after** the course?"

Paired t-Test (Dependent t-Test)

A **paired t-test** is used when we compare **two related (paired) samples**, such as **before-and-after** measurements.

Example: Exam Performance Before & After a Training Program

A professor wants to determine if a **training program** improves student performance. She collects exam scores **before and after** the training for **8 students**:

Student	Before Training	After Training
1	65	70
2	68	74
3	72	78
4	60	65
5	75	80
6	67	71
7	69	74
8	71	77

We will conduct a **paired t-test** to check if there is a significant difference in scores **before and after training**.

Step 1: State the Hypotheses

- **Null Hypothesis (H0)**: The training program has no effect on scores. $H_0: \mu_d = 0$
- **Alternative Hypothesis (H1)**: The training program **improves** scores (one-tailed test). $H_1: \mu_d > 0$

Where μ_d is the mean difference between **before** and **after** scores.

Step 2: Choose the Significance Level

- $\alpha = 0.05$ (5%)

Step 3: Compute the Test Statistic (t-score)

The formula for a **paired t-test** is:

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$

Where:

- $\bar{d}$ = mean of the differences $(\text{After} - \text{Before})$
- s_d = standard deviation of the differences
- n = sample size

Let's calculate the **t-score**.

The **calculated t-score** is **21.0**, which is very large, indicating a strong difference between the before and after scores.

Step 4: Find the Critical Value (t-critical)

For a **one-tailed t-test** at α=0.05 with **degrees of freedom (df) = n - 1 = 8 - 1 = 7**, we find the **critical t-value**. Let's calculate it.

The **critical t-value** for a one-tailed test at α=0.05 with 7 degrees of freedom is **1.89**.

Step 5: Make a Decision

- Our **calculated t-score** is **21.0**.
- The **critical t-value** is **1.89**.
- Since t>tcritical (**21.0 > 1.89**), we **reject the null hypothesis**.

Step 6: Conclusion

At the **5% significance level**, there is **strong evidence** to conclude that the **training program significantly improves student performance**.

one-sample t-test:

A one-sample t-test is used when we want to compare the mean of a sample to a known population mean when the population standard deviation is unknown.

Example: Testing Average Exam Scores

A professor believes that the average exam score of students in a university is **75**. To test this claim, a sample of **12 students** is taken, and their exam scores are recorded:

Sample Data (exam scores)

80, 85, 78, 92, 88, 76, 81, 79, 74, 77, 83, 86

The professor sets a significance level of **0.05**.

Step 1: State the Hypotheses

- **Null Hypothesis (H0)**: The average exam score is **75**. $H0: \mu = 75$
- **Alternative Hypothesis (H1)**: The average exam score is **not equal to 75**. $H1: \mu \neq 75$ *(This is a* **two-tailed** *test because we are testing for any difference, not just greater or smaller.)*

Step 2: Choose the Significance Level

- $\alpha = 0.05$ (5%)

Step 3: Compute the Test Statistic (t-score)

The formula for a **one-sample t-test** is:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Where:

- $\bar{x}$ = sample mean
- μ = population mean (75)
- s = sample standard deviation
- n = sample size (12)

Let's calculate the **t-score**.

The **calculated t-score** is **4.26**.

Step 4: Find the Critical Value (t-critical)

For a **two-tailed t-test** at $\alpha = 0.05$ with **degrees of freedom (df) = n - 1 = 12 - 1 = 11**, we look up the t-table or use statistical software.

Let’s determine the **critical t-value**.

The **critical t-value** for a two-tailed test at $\alpha = 0.05$ with 11 degrees of freedom is **2.20**.

Step 5: Make a Decision

- Our **calculated t-score** is **4.26**.

- The **critical t-value** is **2.20**.
- Since ∣t∣>tcritical (**4.26 > 2.20**), we **reject the null hypothesis**.

Step 6: Conclusion

At the **5% significance level**, there is **strong evidence** to conclude that the **average exam score is significantly different from 75**.

Two-Sample t-Test (Independent t-Test)

A **two-sample t-test** (also called an **independent t-test**) is used to compare the means of **two independent groups** to determine if there is a significant difference between them.

Example: Comparing Exam Scores of Two Different Classes

A teacher wants to compare the **average exam scores** of two different classes (**Class A and Class B**) to see if there is a significant difference.

Sample Data (Exam Scores):

- **Class A Scores: 85, 88, 90, 93, 87, 92, 89, 91**
- **Class B Scores: 78, 82, 84, 79, 81, 83, 77, 80**

The teacher sets a significance level of **0.05**.

Step 1: State the Hypotheses

- **Null Hypothesis (H0)**: The average scores of Class A and Class B are **equal**. H0:μ1=μ2
- **Alternative Hypothesis (H1)**: The average scores of Class A and Class B are **not equal**. H1 :μ1�=μ2 *(This is a* **two-tailed** *test because we are testing for any difference, not just greater or smaller.)*

Step 2: Choose the Significance Level

- α=0.05 (5%)

Step 3: Compute the Test Statistic (t-score)

The formula for a **two-sample t-test** (assuming unequal variances) is:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Where:

- $\bar{x}_1, \bar{x}_2$ = sample means of groups
- s_1, s_2 = sample standard deviations of groups
- n_1, n_2 = sample sizes

Let's calculate the **t-score**.

The **calculated t-score** is **6.93**, which is quite large, indicating a strong difference between the two groups.

Step 4: Find the Critical Value (t-critical)

For a **two-tailed t-test** at $\alpha = 0.05$, we determine the degrees of freedom (df) using the **Welch-Satterthwaite equation**:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

Let's calculate the **degrees of freedom** and find the **critical t-value**. [>-]

The **degrees of freedom (df)** is approximately **13.90**, and the **critical t-value** for a two-tailed test at $\alpha = 0.05$ is **±2.15**.

Step 5: Make a Decision

- Our **calculated t-score** is **6.93**.
- The **critical t-value** is **±2.15**.
- Since $|t| > t_{critical}$ (**6.93 > 2.15**), we **reject the null hypothesis**.

Step 6: Conclusion

At the **5% significance level**, there is **strong evidence** to conclude that the **average exam scores of Class A and Class B are significantly different**.

Example of Chi-Square Test in Hypothesis Testing

The **Chi-Square Test** is used to determine if there is a significant association between two categorical variables.

Scenario: Customer Preference for Product Packaging

A company wants to know if customer preference for product packaging (A, B, or C) is **independent of gender** (Male or Female).

Step 1: Define Hypotheses

- **Null Hypothesis (H0)**: There is no relationship between gender and packaging preference (they are independent).
- **Alternative Hypothesis (HA)**: There is a relationship between gender and packaging preference (they are dependent).

Step 2: Collect Data (Observed Frequencies Table)

Packaging Type	Male	Female	Total
A	30	40	70
B	50	30	80
C	20	30	50
Total	100	100	200

Step 3: Compute Expected Frequencies

Expected frequency for each cell:

TotalE=Grand Total(RowTotal)×(ColumnTotal)

For example, expected frequency for **Male, A**:

EMale,A=200(70×100)=35

Following this method, we calculate all expected frequencies:

Packaging Type	Male (Expected)	Female (Expected)
A	35	35
B	40	40
C	25	25

Step 4: Compute Chi-Square Statistic

χ2=∑E(O−E)2

Where:

- O = observed frequency
- E = expected frequency

For **Male, A**:

35(30−35)2=3525=0.71

Computing for all cells:

Packaging Type	Observed (O)	Expected (E)	(O−E)2/E
A, Male	30	35	0.71
A, Female	40	35	0.71
B, Male	50	40	2.5
B, Female	30	40	2.5
C, Male	20	25	1.0
C, Female	30	25	1.0

Summing these:

χ2=0.71+0.71+2.5+2.5+1.0+1.0=8.42

Step 5: Determine the Critical Value

- **Degrees of Freedom (df)**:

(df)=(rows−1)×(columns−1)=(3−1)×(2−1)=2

- **Significance Level (α)**: 0.05
- **Critical Value from Chi-Square Table (df = 2, α=0.05)**: **5.99**

Step 6: Decision Rule

- If χ^2 > critical value, reject H0 (there is an association).
- If $\chi^2 \leq$ critical value, fail to reject H0 (no evidence of an association).

Since **8.42 > 5.99**, we **reject** the null hypothesis.

Step 7: Conclusion

There is **a significant association** between gender and packaging preference. This suggests that men and women **do not** prefer packaging in the same way.

Simple Example of Chi-Square Test

Scenario:

A teacher wants to know if students' performance (Pass/Fail) is independent of the teaching method used (Traditional/Modern).

Step 1: Define Hypotheses

- **Null Hypothesis (H0)**: There is no relationship between teaching method and student performance (they are independent).
- **Alternative Hypothesis (HA)**: There is a relationship between teaching method and student performance (they are dependent).

Step 2: Observed Data (Contingency Table)

Performance	Traditional Method	Modern Method	Total
Pass	30	50	80
Fail	20	10	30
Total	50	60	110

Step 3: Compute Expected Values

Formula:

$E=\text{Grand Total}(\text{Row Total}\times\text{Column Total})$

For **Pass, Traditional**:

$E\text{Pass,Traditional}=110(80\times50)=36.36$

For **Pass, Modern**:

$E\text{Pass,Modern}=110(80\times60)=43.64$

For **Fail, Traditional**:

$E\text{Fail,Traditional}=110(30\times50)=13.64$

For **Fail, Modern**:

$E\text{Fail,Modern}=110(30\times60)=16.36$

Performance	Traditional (Expected)	Modern (Expected)
Pass	36.36	43.64
Fail	13.64	16.36

Step 4: Compute Chi-Square Statistic

$\chi2=\sum E(O-E)2$

For each cell:

$36.36(30-36.36)2=36.3640.45=1.11$ $43.64(50-43.64)2=43.6440.45=0.93$ $13.64(20-13.64)2=13.6440.45=2.97$ $16.36(10-16.36)2=16.3640.45=2.47$

Summing these:

$\chi2=1.11+0.93+2.97+2.47=7.48$

Step 5: Compare with Critical Value

- **Degrees of Freedom** = $(2-1)\times(2-1)=1$
- **Significance Level** = 0.05
- **Chi-Square Critical Value (df = 1, α = 0.05) = 3.84**

Since **7.48 > 3.84**, we **reject** H0.

Step 6: Conclusion

There is a significant relationship between **teaching method and student performance**. The modern teaching method may be more effective.

Example of Type I Error

Imagine a **medical test** for a new drug.

- **H0 (Null Hypothesis):** The drug has **no effect**.
- **H1 (Alternative Hypothesis):** The drug **works**.
- **Type I Error:** The test **incorrectly concludes** that the drug is effective, even though it **actually has no effect**.

Consequence: The drug gets approved, but it doesn't work, wasting money and possibly harming patients.

Probability of Type I Error: Significance Level (α)

The **probability** of making a Type I Error is controlled by the **significance level** (α):

- If **$\alpha=0.05$**, there is a **5% chance** of wrongly rejecting a true null hypothesis.
- Lowering α (e.g., to **0.01**) **reduces** the chance of a Type I Error but **increases** the chance of a Type II Error.

Key Takeaways

- **Type I Error: False Positive** (Reject H0 when it is actually true).
- **Controlled by α** (typically 0.05 or 5%).
- **Example:** Declaring a person guilty when they are actually innocent.

Type II Error (False Negative Error)

A **Type II Error** occurs when we **fail to reject a false null hypothesis** (H0). This means we **incorrectly conclude** that there is no effect or relationship when there actually is one.

Example of Type II Error

Imagine a **medical test** for a new drug:

- **H0 (Null Hypothesis):** The drug has **no effect**.
- **H1 (Alternative Hypothesis):** The drug **works**.
- **Type II Error:** The test **incorrectly concludes** that the drug has no effect, even though it **actually works**.

Consequence: A useful drug is **not approved**, and patients miss out on a beneficial treatment.

Probability of Type II Error: Beta (β) and Power of the Test

- The **probability of making a Type II Error** is denoted by **β**.
- The **Power of a test** is $1-\beta$ and represents the ability to detect a real effect.

- To **reduce Type II Errors**, we can:
  - **Increase the sample size**
  - **Increase the significance level (α)**
  - **Use a more powerful test**

Comparison of Type I vs. Type II Errors

	Type I Error	Type II Error
Definition	Rejecting H0 when it is actually **true**	Failing to reject H0 when it is actually **false**
Also Called	**False Positive**	**False Negative**
Example	Approving a drug that **does not work**	Rejecting a drug that **actually works**
Controlled by	α (significance level)	β (related to test power)
Consequence	Acting on a false effect	Missing a real effect

Key Takeaways

- **Type I Error (False Positive):** Incorrectly **rejecting** a true null hypothesis.
- **Type II Error (False Negative):** Incorrectly **failing to reject** a false null hypothesis.
- There is often a **trade-off**: Reducing one increases the other.

Type I and Type II Errors: Understanding the Difference

Both **Type I** and **Type II Errors** occur in **hypothesis testing** and represent incorrect decisions made based on sample data.

Definition & Examples

Error Type	Definition	Example: Medical Test	Example: Court Trial
Type I Error (False Positive)	Rejecting a true null hypothesis (H0)	A test **incorrectly detects a disease** in a healthy person.	Convicting an **innocent** person.
Type II Error (False Negative)	Failing to reject a false null hypothesis (H0)	A test **fails to detect a disease** in a sick person.	Letting a **guilty** person go free.

Graphical Representation

Imagine a normal distribution where:

- **Type I Error (False Positive)** occurs when you incorrectly **reject H0** (thinking there is an effect when there isn't).
- **Type II Error (False Negative)** occurs when you fail to reject H0 even though there is an actual effect.

A **lower significance level (α)** reduces Type I Errors but increases Type II Errors, and vice versa.

Trade-Off Between Type I and Type II Errors

There is often a **trade-off** between the two:

- Lowering α (making it harder to reject H0) **reduces Type I Errors** but **increases Type II Errors**.
- Increasing sample size **reduces Type II Errors** without increasing Type I Errors.

Real-World Case Study: COVID-19 Testing & Type I vs. Type II Errors

During the COVID-19 pandemic, diagnostic tests (like **PCR tests**) were widely used to detect infections. However, these tests were not **perfect** and could produce **Type I and Type II Errors**.

Case Study: COVID-19 Testing Errors

1. Setting Up the Hypothesis

- **H0 (Null Hypothesis)**: The person **does not have COVID-19**.
- **H1 (Alternative Hypothesis)**: The person **has COVID-19**.

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2. Type I Error (False Positive) in COVID-19 Testing

- **What happens?** The test **incorrectly detects COVID-19** in a person who is actually healthy.
- **Consequences:**
  - The person is **wrongly isolated** and may face **mental stress**.
  - Resources (hospital beds, medicines) may be **wasted**.
  - False positives can **spread fear** in the community.

Example: A healthy individual takes a test before traveling, gets a **false positive**, and is **denied entry** to another country.

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3. Type II Error (False Negative) in COVID-19 Testing

- **What happens?** The test **fails to detect COVID-19** in a person who actually has the virus.
- **Consequences:**
  - The infected person **continues to spread** the virus unknowingly.
  - They **miss early treatment**, leading to **severe health complications**.
  - The pandemic worsens due to **silent carriers**.

Real-World Case Study: Type I & Type II Errors in COVID-19 Testing

Background

During the COVID-19 pandemic, diagnostic tests (like **PCR tests** and **rapid antigen tests**) were used worldwide to detect infections. However, these tests were not **100% accurate** and sometimes produced **Type I (False Positive) and Type II (False Negative) Errors**.

Understanding Type I and Type II Errors in COVID-19 Testing

Hypothesis Testing	Reality: No COVID-19	Reality: Has COVID-19
Test Result: Positive	**Type I Error (False Positive)** – Test wrongly detects COVID-19 in a healthy person.	**Correct Detection (True Positive)**
Test Result: Negative	**Correct Detection (True Negative)**	**Type II Error (False Negative)** – Test fails to detect COVID-19 in an infected person.

Case Study: COVID-19 Testing in a Global Pandemic

1. Type I Error (False Positive) in COVID-19 Testing

What happens?

- The test **incorrectly detects COVID-19** in a person who is actually healthy.

Real-World Example:

- In some **travel restrictions**, a **false positive test** prevented people from boarding flights or entering countries, even though they were healthy.
- In hospitals, **patients without COVID-19 were wrongly isolated**, taking up medical resources unnecessarily.

Consequences:

- The person may face **unnecessary quarantine or isolation**.
- Economic loss due to **missed work, travel, or social events**.
- Wastage of medical resources and **unnecessary psychological stress**.

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2. Type II Error (False Negative) in COVID-19 Testing

What happens?

- The test **fails to detect COVID-19** in an infected person.

Real-World Example:

- Some **rapid antigen tests** had lower sensitivity and produced **false negatives**, meaning **infected individuals mistakenly believed they were COVID-free**.
- This led to **infected people unknowingly spreading the virus** in workplaces, public spaces, and hospitals.

Consequences:

- **Increased transmission** of the virus, leading to **larger outbreaks**.
- **Delayed treatment** for infected individuals, increasing **hospitalization rates**.
- **False sense of security**, causing people to ignore necessary precautions.

Balancing the Trade-Off:

Governments and health organizations had to balance:

- **Lowering Type I Errors** (avoiding unnecessary quarantines & false alarms).
- **Lowering Type II Errors** (ensuring infected people were detected).

This was achieved by: **Using both PCR & rapid tests** (PCR for accuracy, antigen tests for quick screening).
Retesting when symptoms were present but results were negative.
Improving test sensitivity & specificity to reduce both error types.