UNIT - II

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Subject :- TOC
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REGULAR EXPRESSION (RE)

* Regular Expression/set :-

→ "The set of string accepted by finite automata is known as Regular Language."

operator :- This language can also be described in a compact form using a set of operators.

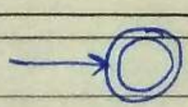
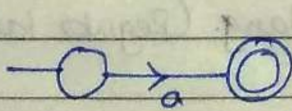
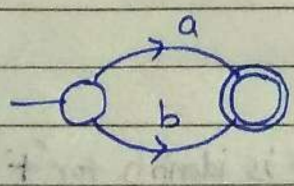
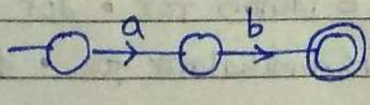
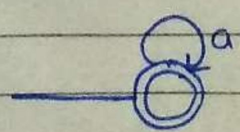
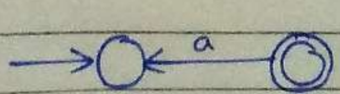
These operators are :-

1) + , union operator
2) • , Concatenation operator
3) * , star / closure operator.

Regular Expression :-

Def :- "An expression written using set of operators (+, •, *) and describing a regular language is known as Regular Expression."

The Regular expression for some basic Automata.

Automata	Language	Regular expression
	$\{\epsilon\}$	$R \cdot E = \epsilon$
	$\{a\}$	$R.E. = a$
	$\{a, b\}$	$R.E. = a+b$
	$\{ab\}$	$R.E = a \cdot b$
	$\{\epsilon, a, aa, aaa \ldots\}$	$R.E = a^*$
	ϕ	$R.E = \phi$

* Precedence of Operators :

→ operator are associated with operand in particular order. The Precedence of operators for regular expression is as follow :

1) The star operator has the highest Precedence
2) The concatenation or "dot" operator comes next in precedence
3) Union or "+" operators comes last in the Precedence.

* Algebraic laws for Regular Expression :

→ There are number of laws for algebraic laws :

1) Associativity and commutativity.
2) Identities and annihilators.
3) The idempotent law.
4) laws of involving closures. 5) Distributive law.

1) Associativity & Commutativity

- Commutative laws for union says that the union of two regular languages can be taken in any order.
- For any two lang. L and M.

$$L+M = M+L$$

- The Associative law holds for union of two lang. (Regular lang)

$$(L+M)+N = L+(M+N)$$

2) Identities and annihilator :

ϵ is identity and ϕ is annihilator

There are 3 law

1) $\phi + L = L + \phi = L$ ϕ is identity for '+'
2) $\epsilon \cdot L = L \cdot \epsilon = L$ ϵ identity for • dot
3) $\phi \cdot L = L \cdot \phi = \phi$ is annihalator for • dot

3) Distributive law :

1) Left distributive law of concatenation over union

$$L(M+N) = LM+LN.$$

2) Right distributive law of concatenation

$$(M+N)L = ML+NL$$

4. Idempotent Law :-

It says that the union of two identical expression can be replaced by one copy of expression.

$$L + L = L$$

5. Laws of Involving closures.

These laws include :-

1. $(L^*)^* = L$
2. $\phi^* = \epsilon$
3. $\epsilon^* = \epsilon$
4. $L^* = LL^* = L^*L$.
5. $L^* = L^* + \epsilon$

* Pumping Lemma for Regular Language :-

Def :- "Let L be a regular language and $M = \{Q, \Sigma, \delta, q_0, F\}$ be a finite automata with n-states. Language L is accepted by M. Let $w \in L$ and $|w| \geq n$ then w can be written as xyz where.

i) $|y| > 0$ ii) $|xy| \leq n$

iii) $xy^iz \in L$ for all $i \geq 0$ here y^i denotes that y is repeated or pumped i times"

- pumping lemma should be used to establish that a given language is not regular

* Closure properties of Regular Language :-

"If an operation on regular lang. generates a Regular lang. then we say that "the class of regular lang. is closed under above operation".

Some of the closure properties :-

1) Union
2) Difference
3) Concatenation
4) Intersection
5) Complementation
6) Kleene star.

* Regular Language is closed under union :-

→ Let $M_1 = \{S, \Sigma, \delta_1, S_0, F)$ and
$M_2 = (Q, \Sigma, \delta_2, q_0, G)$ be two given automata.

To prove closure property, we need M_3 variable which accept every string accepted either by M_1 or M_2.

$$\therefore \; L(M_3) = L(M_1) \cup L(M_2)$$

* Regular lang. is closed under Concatenation :-

→

$$L(M_3) = L(M_1) \cdot L(M_2)$$

* Regular lang is closed under Kleene star (closure)

→

$$L(M_2) = L(M_1)^*$$

* Difference :- $L(M_3) = L(M_1) - L(M_2)$

* Intersection :- $L(M_3) = L(M_1) \cap L(M_2)$

* Complementation :- $L(M_1) = \overline{L(M_1)}$

* Reversal :- $L = \{aab, abb, aaa)$
then $L^R = \{baa, bba, aaa\}$;

* Show that the set ($L = \{b^{i^2} \mid i > 1\}$ is not regular.

→ We have to prove that the lang $L = b^{i^2}$ is not regular. This lang is such that nos of b's is alway a perfect square.

For example,

if we take $i = 1$

$L = b^{1^2} = b$ then length $= 1^2$

$= b^{2^2} = bbbb$ then length $= 2^2$

and so on.

Now let us consider, $L = b^{n^2}$ where length n^2

it is denoted by Z

$|Z| = n^2$

By pumping lemma,

$Z = uvw$ where $1 \le |v| \le n$.

As $Z = uv^i w$ where $i = 1$.

Now we will pump v i.e. $i = 2$

As we made $i = 2$ we have added one n^2 $1 \le |v| \le 2$

$n^2 + 1 \le |uvw| \le n + n^2$

Thus, string lies bet^n 2 consecutive Perfect square. But string is not perfect square, Hence we can say the given lang. is not regular.

<u>For example</u> :-

$L = b^{i^2}$

$i = 2$

$L = bbbb$

$L = uvw$

Assume $uvw = bbbb$

$u = b$

$v = bb$

$w = b$.

By pumping lemma, even we pump v i.e increase v. then lang. should show the length as perfect square.

$= uvw$

$= uv \cdot v \cdot w$ ∴ $u = b$, $v = bb$, $v = bb$, $w = b$

$= bbbbbb$.

The length of b is not Perfect square. so it is not regular.

* Prove / Disprove that the Language L given by
$L = \{a^m a^n \mid m \neq n$ and n are +ve integer$\}$ is regular.

→ Let $L = \{a^m a^n \mid m \neq n\}$ be a language
Assume that L is regular lang., Now consider case

i) Case 1 : $z = aaabbbb \in L$.

By pumping lemma if $z = uvw$ then we pump some string to make $z = uv^iw$ then ~~if z~~.
if $z \in L$ then such lang. is called regular.

Let,

$z = \underbrace{a}_{u}\ \underbrace{a\,a\,b\,b}_{v}\ \underbrace{b\,b}_{w}$.

if $z = uv^iw$, and if $i = 2$ then $z = uvvw$.

$z = \underbrace{a}_{u}\ \underbrace{aabb}_{v}\ \underbrace{aabb}_{v}\ \underbrace{bb}_{w}$

$z = a^3 b^2 a^2 b^4 \in a^m b^n \in L$.

ii) Case 2 : Assume $L \in Z$ such that

$z = \underbrace{a}_{u}\ \underbrace{aa}_{v}\ \underbrace{abbbbbb}_{w}$

By pumping lemma,

$z = uv^iw \in L$ for regular lang. If $i = 2$

$z = uvvw$

$z = \underbrace{a}_{u}\ \underbrace{aa}_{v}\ \underbrace{aa}_{v}\ \underbrace{abbbbbb}_{w}$

$z = a^6 b^6 \in a^m a^n$ because $m = n$.

From these case, we get $z \in L$, Thus our assumption of L being regular is wrong.
Hence, given language $\{L = a^m b^n \mid m \neq n\}$ is not regular // Disproved