L6 PDA to CFG Examples

Example 5.6.8 **SPPU - Dec. 13, May 15, May 16 Dec. 16, 9/12 Marks**

Give the CFG generating the language accepted b the following PDA :M = ($\{q_0, q_1\}$, $\{0, 1\}$, $\{z_0, x\}$, δ, q_0, z_0, ϕ) when δ is given below :

$\delta(q_0, 1, z_0) = \{(q_0, xz_0)\}$

$\delta(q_0, 1, x) = \{(q_0, xx)\}$), $\delta(q_0, 0, x) = \{(q_1, x)\}$

$\delta(q_0, \varepsilon, z_0) = \{(q_0, \varepsilon)\}$), $\delta(q_1, 1, x) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 0, z_0) = \{(q_0, z_0)\}$

Solution :

Step 1 : Add productions for the start symbol

$$S \rightarrow [q_0 \, ^{z_0} \, q_0]$$

$$S \rightarrow [q_0 \, ^{z_0} \, q_1]$$

Step 2 : Add productions for $\delta(q_0, 1, z_0) = \{(q_0, xz_0)\}$

$$[q_0 \, ^{z_0} \, q_0] \rightarrow 1 \, [q_0 \, ^{x} \, q_0] \, [q_0 \, ^{z_0} \, q_0]$$

$$[q_0 \, ^{z_0} \, q_0] \rightarrow 1 \, [q_0 \, ^{x} \, q_1] \, [q_1 \, ^{z_0} \, q_0]$$

Example 5.6.8 SPPU - Dec. 13, May 15, May 16 Dec. 16, 9/12 Marks

Give the CFG generating the language accepted b the following PDA : $M = (\{q_0, q_1\}, \{0, 1\}, \{z_0, x\}, \delta, q_0, z_0, \phi)$ when δ is given below :

$\delta(q_0, 1, z_0) = \{(q_0, xz_0)\}$

$\delta(q_0, 1, x) = \{(q_0, xx)\}\}$, $\delta(q_0, 0, x) = \{(q_1, x)\}$

$\delta(q_0, \varepsilon, z_0) = \{(q_0, \varepsilon)\}\}$, $\delta(q_1, 1, x) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 0, z_0) = \{(q_0, z_0)\}$

Solution :

Step 1 : Add productions for the start symbol

$$S \rightarrow [q_0\,{}^{z_0}\,q_0]$$

$$S \rightarrow [q_0\,{}^{z_0}\,q_1]$$

Step 2 : Add productions for $\delta(q_0, 1, z_0) = \{(q_0, xz_0)\}$

$$[q_0\,{}^{z_0}\,q_0] \rightarrow 1\,[q_0\,{}^{x}\,q_0]\,[q_0\,{}^{z_0}\,q_0]$$

$$[q_0\,{}^{z_0}\,q_0] \rightarrow 1\,[q_0\,{}^{x}\,q_1]\,[q_1\,{}^{z_0}\,q_0]$$

$$[q_0\,{}^{z_0}\,q_1] \rightarrow 1\,[q_0\,{}^{x}\,q_0]\,[q_0\,{}^{z_0}\,q_1]$$

$$[q_0\,{}^{z_0}\,q_1] \rightarrow 1\,[q_0\,{}^{x}\,q_1]\,[q_1\,{}^{z_0}\,q_1]$$

Step 3 : Add productions for $\delta(q_0, 1, x) \Rightarrow \{(q_0, xx)\}$

$$[q_0\,{}^{x}\,q_0] \rightarrow 1\,[q_0\,{}^{x}\,q_0]\,[q_0\,{}^{x}\,q_0]$$

$$[q_0\,{}^{x}\,q_0] \rightarrow 1\,[q_0\,{}^{x}\,q_1]\,[q_1\,{}^{x}\,q_0]$$

$$[q_0\,{}^{x}\,q_1] \rightarrow 1\,[q_0\,{}^{x}\,q_0]\,[q_0\,{}^{x}\,q_1]$$

$$[q_0\,{}^{x}\,q_1] \rightarrow 1\,[q_0\,{}^{x}\,q_1]\,[q_1\,{}^{x}\,q_1]$$

Step 4 : Add productions for $\delta(q_0, 0, x) \Rightarrow \{(q_1, x)\}$

$$[q_0\,{}^{x}\,q_0] \rightarrow 0\,[q_1\,{}^{x}\,q_0]$$

$$[q_0\,{}^{x}\,q_1] \rightarrow 0\,[q_1\,{}^{x}\,q_1]$$

Step 5 : Add productions for $\delta(q_0, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

$$[q_0\,{}^{z_0}\,q_1] \rightarrow \varepsilon$$

Step 6 : Add production for $\delta(q_1, 1, x) \Rightarrow \{(q_1, \varepsilon)\}$

$$[q_1\,{}^{x}\,q_1] \rightarrow 1$$

Step 7 : Add productions for $\delta(q_1, 0, z_0) \Rightarrow \{(q_0, z_0)\}$

$$[q_1\,{}^{z_0}\,q_0] \Rightarrow 0\,[q_0\,{}^{z_0}\,q_0]$$

$$[q_1\,{}^{z_0}\,q_1] \Rightarrow 0\,[q_0\,{}^{z_0}\,q_1]$$

Example 5.6.9 SPPU - May 14, Dec. 15, 8/12 Marks

Consider the PDA with the following moves :

$\delta(q_0, a, z_0) = \{(q_0, az_0)\}$, $\delta(q_0, a, a) = \{(q_0, aa)\}$

$\delta(q_0, b, a) = \{(q_1, \varepsilon)\}$, $\delta(q_1, b, a) = \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

Obtain CFG equivalent to PDA.

Solution :

Step 1 : Add productions for the start symbol.

$$S \rightarrow [q_0 \; {}^{z_0} \; q_0]$$

$$S \rightarrow [q_0 \; {}^{z_0} \; q_1]$$

Step 2 : Add productions for $(q_0, a, a) = \{(q_0, aa)\}$

$$[q_0 \; {}^{a} \; q_0] \rightarrow a \, [q_0 \; {}^{a} \; q_0] \, [q_0 \; {}^{a} \; q_0]$$

$$[q_0 \; {}^{a} \; q_0] \rightarrow a \, [q_0 \; {}^{a} \; q_1] \, [q_1 \; {}^{a} \; q_0]$$

$$[q_0 \; {}^{a} \; q_1] \rightarrow a \, [q_0 \; {}^{a} \; q_0] \, [q_0 \; {}^{a} \; q_1]$$

$$[q_0 \; {}^{a} \; q_1] \rightarrow a \, [q_0 \; {}^{a} \; q_1] \, [q_1 \; {}^{a} \; q_1]$$

Step 3 : Add productions for $\delta(q_0, b, a) = \{(q_1, \varepsilon)\}$

$$[q_0 \; {}^{a} \; q_1] \rightarrow b$$

Step 4 : Add productions for $\delta(q_1, b, a) = \{(q_1, \varepsilon)\}$

$$[q_1 \; {}^{a} \; q_1] \rightarrow b$$

Step 5 : Add productions for $\delta(q_1, \varepsilon, z_0) \rightarrow \{(q_1, \varepsilon)\}$

$$[q_1 \; {}^{z_0} \; q_1] \rightarrow \varepsilon$$

Example 5.6.10

For the PDA

$(\{ q_0, q_1 \}, \{ 0, 1 \}, \{ 0, 1, z_0 \}, \delta, q_0, z_0, \phi)$ where δ is

$\delta(q_0, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

$\delta(q_0, 0, z_0) = \{(q_0, 0z_0)\}$

$\delta(q_0, 0, 0) = \{(q_0, 00)\}$

$\delta(q_0, 1, 0) = \{(q_0, 10)\}$

$\delta(q_0, 1, 1) = \{(q_0, 11)\}$

$\delta(q_0, 0, 1) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 0, 1) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

Example 5.6.10

For the PDA

$(\{q_0, q_1\}, \{0, 1\}, \{0, 1, z_0\}, \delta, q_0, z_0, \phi)$ where δ is

$\delta(q_0, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

$\delta(q_0, 0, z_0) = \{(q_0, 0z_0)\}$

$\delta(q_0, 0, 0) = \{(q_0, 00)\}$

$\delta(q_0, 1, 0) = \{(q_0, 10)\}$

$\delta(q_0, 1, 1) = \{(q_0, 11)\}$

$\delta(q_0, 0, 1) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 0, 1) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

Sr. No.	PDA transition	Corresponding productions
1.	Productions due to start symbol S.	$S \rightarrow [q_0 \overset{z_0}{} q_0]$
		$S \rightarrow [q_0 \overset{z_0}{} q_1]$
2.	$\delta(q_0, \varepsilon, z_0) = (q_1, \varepsilon)$	$[q_0 \overset{z_0}{} q_1] \rightarrow \varepsilon$
3.	$\delta(q_0, 0, z_0) = (q_0, 0z_0)$	$[q_0 \overset{z_0}{} q_0] \rightarrow 0 [q_0 \overset{0}{} q_0] [q_0 \overset{z_0}{} q_0]$
		$[q_0 \overset{z_0}{} q_0] \rightarrow 0 [q_0 \overset{0}{} q_1] [q_1 \overset{z_0}{} q_0]$
		$[q_0 \overset{z_0}{} q_1] \rightarrow 0 [q_0 \overset{0}{} q_0] [q_0 \overset{z_0}{} q_1]$
		$[q_0 \overset{z_0}{} q_1] \rightarrow 0 [q_0 \overset{0}{} q_1] [q_1 \overset{z_0}{} q_1]$
	$\delta(q_0, 0, 0) = (q_0, 00)$	$[q_0 \overset{0}{} q_0] \rightarrow 0 [q_0 \overset{0}{} q_0] [q_0 \overset{0}{} q_0]$
		$[q_0 \overset{0}{} q_0] \rightarrow 0 [q_0 \overset{0}{} q_1] [q_1 \overset{0}{} q_0]$
		$[q_0 \overset{0}{} q_1] \rightarrow 0 [q_0 \overset{0}{} q_0] [q_0 \overset{0}{} q_1]$
		$[q_0 \overset{0}{} q_1] \rightarrow 0 [q_0 \overset{0}{} q_1] [q_1 \overset{0}{} q_1]$
	$\delta(q_0, 1, 0) = (q_0, 10)$	$[q_0 \overset{0}{} q_0] \rightarrow 1 [q_0 \overset{1}{} q_0] [q_0 \overset{0}{} q_0]$
		$[q_0 \overset{0}{} q_0] \rightarrow 1 [q_0 \overset{1}{} q_1] [q_1 \overset{0}{} q_0]$
		$[q_0 \overset{0}{} q_1] \rightarrow 1 [q_0 \overset{1}{} q_0] [q_0 \overset{0}{} q_1]$
		$[q_0 \overset{0}{} q_1] \rightarrow 1 [q_0 \overset{1}{} q_1] [q_1 \overset{0}{} q_1]$
	$\delta(q_0, 1, 1) = (q_0, 11)$	$[q_0 \overset{1}{} q_0] \rightarrow 1 [q_0 \overset{1}{} q_0] [q_0 \overset{1}{} q_0]$
		$[q_0 \overset{1}{} q_0] \rightarrow 1 [q_0 \overset{1}{} q_1] [q_1 \overset{1}{} q_0]$

Example 5.6.10

For the PDA

$(\{q_0, q_1\}, \{0, 1\}, \{0, 1, z_0\}, \delta, q_0, z_0, \phi)$ where δ is

$\delta(q_0, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

$\delta(q_0, 0, z_0) = \{(q_0, 0z_0)\}$

$\delta(q_0, 0, 0) = \{(q_0, 00)\}$

$\delta(q_0, 1, 0) = \{(q_0, 10)\}$

$\delta(q_0, 1, 1) = \{(q_0, 11)\}$

$\delta(q_0, 0, 1) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 0, 1) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

Sr. No.	PDA transition	Corresponding productions
1.	Productions due to start symbol S.	$S \rightarrow [q_0\ z_0\ q_0]$
		$S \rightarrow [q_0\ z_0\ q_1]$
2.	$\delta(q_0, \varepsilon, z_0) = (q_1, \varepsilon)$	$[q_0\ z_0\ q_1] \rightarrow \varepsilon$
3.	$\delta(q_0, 0, z_0) = (q_0, 0z_0)$	$[q_0\ z_0\ q_0] \rightarrow 0\,[q_0\ 0\ q_0]\,[q_0\ z_0\ q_0]$
		$[q_0\ z_0\ q_0] \rightarrow 0\,[q_0\ 0\ q_1]\,[q_1\ z_0\ q_0]$
		$[q_0\ z_0\ q_1] \rightarrow 0\,[q_0\ 0\ q_0]\,[q_0\ z_0\ q_1]$
		$[q_0\ z_0\ q_1] \rightarrow 0\,[q_0\ 0\ q_1]\,[q_1\ z_0\ q_1]$
	$\delta(q_0, 0, 0) = (q_0, 00)$	$[q_0\ 0\ q_0] \rightarrow 0\,[q_0\ 0\ q_0]\,[q_0\ 0\ q_0]$
		$[q_0\ 0\ q_0] \rightarrow 0\,[q_0\ 0\ q_1]\,[q_1\ 0\ q_0]$
		$[q_0\ 0\ q_1] \rightarrow 0\,[q_0\ 0\ q_0]\,[q_0\ 0\ q_1]$
		$[q_0\ 0\ q_1] \rightarrow 0\,[q_0\ 0\ q_1]\,[q_1\ 0\ q_1]$
	$\delta(q_0, 1, 0) = (q_0, 10)$	$[q_0\ 0\ q_0] \rightarrow 1\,[q_0\ 1\ q_0]\,[q_0\ 0\ q_0]$
		$[q_0\ 0\ q_0] \rightarrow 1\,[q_0\ 1\ q_1]\,[q_1\ 0\ q_0]$
		$[q_0\ 0\ q_1] \rightarrow 1\,[q_0\ 1\ q_0]\,[q_0\ 0\ q_1]$
		$[q_0\ 0\ q_1] \rightarrow 1\,[q_0\ 1\ q_1]\,[q_1\ 0\ q_1]$
	$\delta(q_0, 1, 1) = (q_0, 11)$	$[q_0\ 1\ q_0] \rightarrow 1\,[q_0\ 1\ q_0]\,[q_0\ 1\ q_0]$
		$[q_0\ 1\ q_0] \rightarrow 1\,[q_0\ 1\ q_1]\,[q_1\ 1\ q_0]$
		$[q_0\ 1\ q_1] \rightarrow 1\,[q_0\ 1\ q_0]\,[q_0\ 1\ q_1]$
		$[q_0\ 1\ q_1] \rightarrow 1\,[q_0\ 1\ q_1]\,[q_1\ 1\ q_1]$
	$\delta(q_0, 0, 1) = (q_1, \varepsilon)$	$[q_0\ 1\ q_1] \rightarrow 0$
	$\delta(q_1, 0, 1) = (q_1, \varepsilon)$	$[q_1\ 1\ q_1] \rightarrow 0$
	$\delta(q_1, 0, 0) = (q_1, \varepsilon)$	$[q_1\ 0\ q_1] \rightarrow 0$
	$\delta(q_1, \varepsilon, z_0) = (q_1, \varepsilon)$	$[q_1\ z_0\ q_1] \rightarrow \varepsilon$

Sr. No.	PDA transition	Corresponding productions
1.	Productions due to start symbol S.	$S \rightarrow [q_0\,^{z_0}\,q_0]$
		$S \rightarrow [q_0\,^{z_0}\,q_1]$
2.	$\delta(q_0, \varepsilon, z_0) = (q_1, \varepsilon)$	$[q_0\,^{z_0}\,q_1] \rightarrow \varepsilon$
3.	$\delta(q_0, 0, z_0) = (q_0, 0z_0)$	$[q_0\,^{z_0}\,q_0] \rightarrow 0\,[q_0\,^{0}\,q_0]\,[q_0\,^{z_0}\,q_0]$
		$[q_0\,^{z_0}\,q_0] \rightarrow 0\,[q_0\,^{0}\,q_1]\,[q_1\,^{z_0}\,q_0]$
		$[q_0\,^{z_0}\,q_1] \rightarrow 0\,[q_0\,^{0}\,q_0]\,[q_0\,^{z_0}\,q_1]$
		$[q_0\,^{z_0}\,q_1] \rightarrow 0\,[q_0\,^{0}\,q_1]\,[q_1\,^{z_0}\,q_1]$
	$\delta(q_0, 0, 0) = (q_0, 00)$	$[q_0\,^{0}\,q_0] \rightarrow 0\,[q_0\,^{0}\,q_0]\,[q_0\,^{0}\,q_0]$
		$[q_0\,^{0}\,q_0] \rightarrow 0\,[q_0\,^{0}\,q_1]\,[q_1\,^{0}\,q_0]$
		$[q_0\,^{0}\,q_1] \rightarrow 0\,[q_0\,^{0}\,q_0]\,[q_0\,^{0}\,q_1]$
		$[q_0\,^{0}\,q_1] \rightarrow 0\,[q_0\,^{0}\,q_1]\,[q_1\,^{0}\,q_1]$
	$\delta(q_0, 1, 0) = (q_0, 10)$	$[q_0\,^{0}\,q_0] \rightarrow 1\,[q_0\,^{1}\,q_0]\,[q_0\,^{0}\,q_0]$
		$[q_0\,^{0}\,q_0] \rightarrow 1\,[q_0\,^{1}\,q_1]\,[q_1\,^{0}\,q_0]$
		$[q_0\,^{0}\,q_1] \rightarrow 1\,[q_0\,^{1}\,q_0]\,[q_0\,^{0}\,q_1]$
		$[q_0\,^{0}\,q_1] \rightarrow 1\,[q_0\,^{1}\,q_1]\,[q_1\,^{0}\,q_1]$
	$\delta(q_0, 1, 1) = (q_0, 11)$	$[q_0\,^{1}\,q_0] \rightarrow 1\,[q_0\,^{1}\,q_0]\,[q_0\,^{1}\,q_0]$
		$[q_0\,^{1}\,q_0] \rightarrow 1\,[q_0\,^{1}\,q_1]\,[q_1\,^{1}\,q_0]$

Sr. No.	PDA transition	Corresponding productions
		$[q_0\,^{1}\,q_1] \rightarrow 1\,[q_0\,^{1}\,q_0]\,[q_0\,^{1}\,q_1]$
		$[q_0\,^{1}\,q_1] \rightarrow 1\,[q_0\,^{1}\,q_1]\,[q_1\,^{1}\,q_1]$
	$\delta(q_0, 0, 1) = (q_1, \varepsilon)$	$[q_0\,^{1}\,q_1] \rightarrow 0$
	$\delta(q_1, 0, 1) = (q_1, \varepsilon)$	$[q_1\,^{1}\,q_1] \rightarrow 0$
	$\delta(q_1, 0, 0) = (q_1, \varepsilon)$	$[q_1\,^{0}\,q_1] \rightarrow 0$
	$\delta(q_1, \varepsilon, z_0) = (q_1, \varepsilon)$	$[q_1\,^{z_0}\,q_1] \rightarrow \varepsilon$

Simplification of grammar :

We can rename the variables as given below.

$[q_0\,^{z_0}\,q_0] - A$, $[q_0\,^{z_0}\,q_1] - B$, $[q_1\,^{z_0}\,q_0] - C$, $[q_1\,^{z_0}\,q_1] - D$

$[q_0\,^{z_0}\,q_0] - E$, $[q_0\,^{0}\,q_1] - F$, $[q_1\,^{0}\,q_0] - G$, $[q_1\,^{0}\,q_1] - H$

$[q_0\,^{1}\,q_0] - I$, $[q_0\,^{1}\,q_1] - J$, $[q_1\,^{1}\,q_0] - K$, $[q_1\,^{1}\,q_1] - L$

With the above substitutions, the resulting set of productions can be written as :

$S \rightarrow A \mid B$	$B \rightarrow \varepsilon$
$A \rightarrow 0EA \mid 0FC$	$B \rightarrow 0EB \mid 0FD$
$E \rightarrow 0EE \mid 0FG$	$F \rightarrow 0EF \mid 0FH$
$E \rightarrow 1IE \mid 1JG$	$F \rightarrow 1IF \mid 1JH$
$I \rightarrow 1II \mid 1JK$	$J \rightarrow 1IJ \mid 1JL$
$J \rightarrow 0$	$L \rightarrow 0$
$H \rightarrow 0$	$D \rightarrow \varepsilon$

Simplification of grammar :

We can rename the variables as given below.

$[q_0 \, Z_0 \, q_0] - A$, $[q_0 \, Z_0 \, q_1] - B$, $[q_1 \, Z_0 \, q_0] - C$, $[q_1 \, Z_0 \, q_1] - D$

$[q_0 \, Z_0 \, q_0] - E$, $[q_0 \, 0 \, q_1] - F$, $[q_1 \, 0 \, q_0] - G$, $[q_1 \, 0 \, q_1] - H$

$[q_0 \, 1 \, q_0] - I$, $[q_0 \, 1 \, q_1] - J$, $[q_1 \, 1 \, q_0] - K$, $[q_1 \, 1 \, q_1] - L$

With the above substitutions, the resulting set of productions can be written as :

$S \rightarrow A \mid B$	$B \rightarrow \varepsilon$
$A \rightarrow 0EA \mid 0FC$	$B \rightarrow 0EB \mid 0FD$
$E \rightarrow 0EE \mid 0FG$	$F \rightarrow 0EF \mid 0FH$
$E \rightarrow 1IE \mid 1JG$	$F \rightarrow 1IF \mid 1JH$
$I \rightarrow 1II \mid 1JK$	$J \rightarrow 1IJ \mid 1JL$
$J \rightarrow 0$	$L \rightarrow 0$
$H \rightarrow 0$	$D \rightarrow \varepsilon$

$S \rightarrow A \mid B$ $\quad B \rightarrow \varepsilon$

$A \rightarrow 0EA \mid 0FC$ $\quad B \rightarrow 0EB \mid 0FD$

$E \rightarrow 0EE \mid 0FG$ $\quad F \rightarrow 0EF \mid 0FH$

$E \rightarrow 1IE \mid 1JG$ $\quad F \rightarrow 1IF \mid 1JH$

$I \rightarrow 1II \mid 1JK$ $\quad J \rightarrow 1IJ \mid 1JL$

$J \rightarrow 0$ $\quad L \rightarrow 0$

$H \rightarrow 0$ $\quad D \rightarrow \varepsilon$

1. **Removing ε-productions**

Nullable set = {D, B, S}

ε-productions are removed with resulting set of productions as given below :

$S \rightarrow A \mid B$ $\quad A \rightarrow 0EA \mid 0FC$

$B \rightarrow 0EB \mid 0FD \mid 0E \mid 0F$ $\quad E \rightarrow 0EE \mid 0FG \mid 1IE \mid 1JG$

$F \rightarrow 0EF \mid 0FH \mid 1IF \mid 1JH$ $\quad H \rightarrow 0$

$I \rightarrow 1II \mid 1JK$ $\quad J \rightarrow 1IJ \mid 1JL \mid 0$

$L \rightarrow 0$

2. **Removing non-generating symbols**

Set of productions after elimination of non-generating symbols {A, C, D, E, G, I} is given below :

$S \rightarrow B$

$B \rightarrow 0F$

$F \rightarrow 0FH \mid 1JH$

$H \rightarrow 0$

$J \rightarrow 1JL \mid 0$

$L \rightarrow 0$

S → A | B   B → ε

A → 0EA | 0FC   B → 0EB | 0FD

E → 0EE | 0FG   F → 0EF | 0FH

E → 1IE | 1JG   F → 1IF | 1JH

I → 1II | 1JK   J → 1IJ | 1JL

J → 0   L → 0

H → 0   D → ε

1. Removing ε-productions

Nullable set = {D, B, S}

ε-productions are removed with resulting set of productions as given below :

S → A | B   A → 0EA | 0FC

B → 0EB | 0FD | 0E | 0F   E → 0EE | 0FG | 1IE | 1JG

F → 0EF | 0FH | 1IF | 1JH   H → 0

I → 1II | 1JK   J → 1IJ | 1JL | 0

L → 0

2. Removing non-generating symbols

Set of productions after elimination of non-generating symbols {A, C, D, E, G, I} is given below :

S → B

B → 0F

F → 0FH | 1JH

H → 0

J → 1JL | 0

L → 0

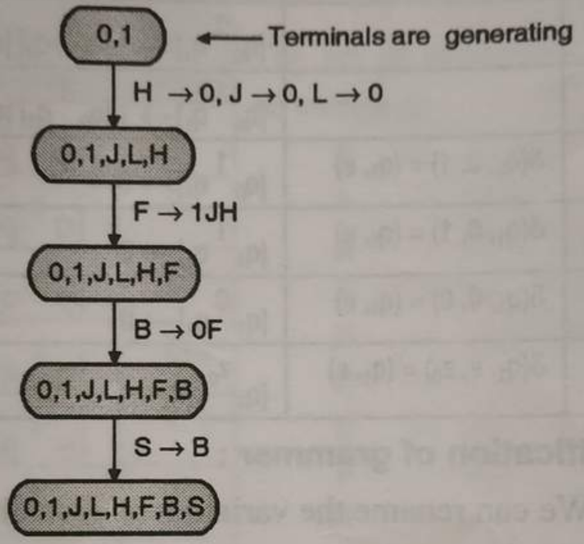

Fig. Ex. 5.6.10

3. The unit production S → B should be removed. The set of productions after elimination of the unit production S → B is given below :

S → 0F   F → 0FH | 1JH

H → 0   J → 1JL | 0

L → 0

Language accepted by the PDA : The language accepted by the PDA is given by :

$$L = \{0^n 1^m 0^{n+m} \mid n, m \geq 1\} \cup \varepsilon$$