

## L2.3 Simplification of CFG (Removal of NULL Productions)

# Simplification of Context Free Grammar

## Removal of Null Productions

In a CFG, a Non-Terminal Symbol 'A' is a nullable variable if there is a production  $A \rightarrow \epsilon$  or there is a derivation that starts at 'A' and leads to  $\epsilon$ . (Like  $A \rightarrow \dots \rightarrow \epsilon$ )

### Procedure for Removal:

- Step 1: To remove  $A \rightarrow \epsilon$ , look for all productions whose right side contains A
- Step 2: Replace each occurrences of 'A' in each of these productions with  $\epsilon$
- Step 3: Add the resultant productions to the Grammar

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Example: Remove Null Productions from the following Grammar

$S \rightarrow ABAC, A \rightarrow aA | \epsilon, B \rightarrow bB | \epsilon, C \rightarrow c$

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$A \rightarrow \epsilon, B \rightarrow \epsilon$



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$$S \rightarrow ABAC, \quad A \rightarrow aA | \epsilon, \quad B \rightarrow bB | \epsilon, \quad C \rightarrow c$$

$$A \rightarrow \epsilon, \quad B \rightarrow \epsilon$$

1) To eliminate  $A \rightarrow \epsilon$

$$S \rightarrow A^R$$

or there is a derivation that starts at  $A$  and leads to  $\epsilon$ . (Like  $A \rightarrow \dots \rightarrow \epsilon$ )

### Procedure for Removal:

- Step 1: To remove  $A \rightarrow \epsilon$ , look for all productions whose right side contains  $A$
- Step 2: Replace each occurrence of ' $A$ ' in each of these productions with  $\epsilon$
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Example: Remove Null Productions from the following Grammar

$$S \rightarrow ABAC, \quad A \rightarrow aA | \epsilon, \quad B \rightarrow bB | \epsilon, \quad C \rightarrow c$$

$$A \rightarrow \epsilon, \quad B \rightarrow \epsilon$$

1) To eliminate  $A \rightarrow \epsilon$

$$S \rightarrow ABAC$$

or there is a derivation that starts at  $A$  and leads to  $\epsilon$ . (Like  $A \rightarrow \dots \rightarrow \epsilon$ )

### Procedure for Removal:

Step 1: To remove  $A \rightarrow \epsilon$ , look for all productions whose right side contains  $A$

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Step 3: Add the resultant productions to the Grammar

Example: Remove Null Productions from the following Grammar

$$S \rightarrow ABAC, \quad A \rightarrow aA | \epsilon, \quad B \rightarrow bB | \epsilon, \quad C \rightarrow c$$

$$A \rightarrow \epsilon, \quad B \rightarrow \epsilon$$

1) To eliminate  $A \rightarrow \epsilon$

$$S \rightarrow \underline{A} \underline{B} \underline{A} C$$

$$S \rightarrow ABC | BAC | BC$$





Step 2: Replace each occurrence of 'A' in each of these productions with  $\epsilon$

Step 3: Add the resultant productions to the Grammar

Example: Remove Null Productions from the following Grammar

$$S \rightarrow ABAC, \quad A \rightarrow aA | \epsilon, \quad B \rightarrow bB | \epsilon, \quad C \rightarrow c$$

$$A \rightarrow \epsilon, \quad B \rightarrow \epsilon$$

1) To eliminate  $A \rightarrow \epsilon$

$$S \rightarrow \underline{A} \underline{B} \underline{A} C$$

$$S \rightarrow ABC | BAC | BC$$

$$A \rightarrow aA$$

$$A \rightarrow a$$

New production:



Step 2: Replace each occurrence of 'A' in each of these productions with  $\epsilon$

Step 3: Add the resultant productions to the Grammar

Example: Remove Null Productions from the following Grammar

$$S \rightarrow ABAC, A \rightarrow aA | \epsilon, B \rightarrow bB | \epsilon, C \rightarrow c$$

$$A \rightarrow \epsilon, B \rightarrow \epsilon$$

1) To eliminate  $A \rightarrow \epsilon$

$$S \rightarrow \underline{A} \underline{B} \underline{A} C$$

$$S \rightarrow ABC | BAC | BC$$

$$A \rightarrow aA$$

$$A \rightarrow a$$

New production:  $S \rightarrow ABAC | ABC | BAC | BC$



$S \rightarrow ABAC, A \rightarrow aA | \epsilon, B \rightarrow bB | \epsilon, C \rightarrow c$

$A \rightarrow \epsilon$  ,  $B \rightarrow \epsilon$

1) To eliminate  $A \rightarrow \epsilon$

$S \rightarrow \underline{A}B\underline{A}C$

$S \rightarrow ABC | BAC | BC$

$A \rightarrow aA$

$A \rightarrow a$

New production:  $S \rightarrow ABAC | ABC | BAC | BC$

$A \rightarrow aA | a, B \rightarrow bB | \epsilon, C \rightarrow c$

$S \rightarrow ABAC, A \rightarrow aA | \epsilon, B \rightarrow bB | \epsilon, C \rightarrow c$

$A \rightarrow \epsilon$  ,  $B \rightarrow \epsilon$

1) To eliminate  $A \rightarrow \epsilon$

$S \rightarrow \underline{A} \underline{B} \underline{A} C$

$S \rightarrow ABC | BAC | BC$

$A \rightarrow aA$

$A \rightarrow a$

New production:  $S \rightarrow ABAC | ABC | BAC | BC$

$A \rightarrow aA | a, B \rightarrow bB | \epsilon, C \rightarrow c$

2) To eliminate  $B \rightarrow \epsilon$

$S \rightarrow ABAC, A \rightarrow aA | \epsilon, B \rightarrow bB | \epsilon, C \rightarrow c$

$A \rightarrow \epsilon$  ,  $B \rightarrow \epsilon$

1) To eliminate  $A \rightarrow \epsilon$

$S \rightarrow \underline{A} \underline{B} \underline{A} C$

$S \rightarrow ABC | BAC | BC$

$A \rightarrow aA$

$A \rightarrow a$

New production:  $S \rightarrow ABAC | ABC | BAC | BC$   
 $A \rightarrow aA | a, B \rightarrow bB | \epsilon, C \rightarrow c$

2) To eliminate  $B \rightarrow \epsilon$

$S \rightarrow AAC |$

$S \rightarrow ABAC, A \rightarrow aA | \epsilon, B \rightarrow bB | \epsilon, C \rightarrow c$

$A \rightarrow \epsilon$  ,  $B \rightarrow \epsilon$

1) To eliminate  $A \rightarrow \epsilon$

$S \rightarrow \underline{A} \underline{B} \underline{A} C$

$S \rightarrow ABC | BAC | BC$

$A \rightarrow aA$

$A \rightarrow a$

New production:  $S \rightarrow ABAC | ABC | BAC | BC$

$A \rightarrow aA | a, B \rightarrow bB | \epsilon, C \rightarrow c$

2) To eliminate  $B \rightarrow \epsilon$

$S \rightarrow AAC | AC |$

$S \rightarrow ABAC, A \rightarrow aA | \epsilon, B \rightarrow bB | \epsilon, C \rightarrow c$

$A \rightarrow \epsilon$  ,  $B \rightarrow \epsilon$

1) To eliminate  $A \rightarrow \epsilon$

$S \rightarrow \underline{A}B\underline{A}C$

$S \rightarrow ABC | BAC | BC$


$A \rightarrow aA$

$A \rightarrow a$

New production:  $S \rightarrow ABAC | ABC | BAC | BC$

$A \rightarrow aA | a, B \rightarrow bB | \epsilon, C \rightarrow c$

2) To eliminate  $B \rightarrow \epsilon$

$S \rightarrow AAC | AC | c$  

1) To eliminate  $A \rightarrow \epsilon$

$$S \rightarrow \underline{A} \underline{B} \underline{A} C$$

$$S \rightarrow ABC \mid BAC \mid BC$$

$$A \rightarrow aA$$

$$A \rightarrow a$$

New production:  $S \rightarrow ABAC \mid ABC \mid BAC \mid BC$

$$A \rightarrow aA \mid a, \quad B \rightarrow bB \mid \epsilon, \quad C \rightarrow c$$

2) To eliminate  $B \rightarrow \epsilon$

$$S \rightarrow AAC \mid AC \mid c, \quad B \rightarrow b$$

New production:



$$S \rightarrow ABC \mid BAC \mid BC$$

$$A \rightarrow aA$$

$$A \rightarrow a$$

New production:  $S \rightarrow ABAC \mid ABC \mid BAC \mid BC$   
 $A \rightarrow aA \mid a$ ,  $B \rightarrow bB \mid \epsilon$ ,  $C \rightarrow C$

2) To eliminate  $B \rightarrow \epsilon$

$$S \rightarrow AAC \mid AC \mid C, \quad B \rightarrow b$$

New production:  $S \rightarrow ABAC \mid ABC \mid BAC \mid BC \mid AAC \mid AC \mid C$   
 $A \rightarrow aA \mid a$   
 $B \rightarrow bB \mid b$

$$S \rightarrow ABC | BAC | BC$$

$$A \rightarrow aA$$

$$A \rightarrow a$$

New production:  $S \rightarrow ABAC | ABC | BAC | BC$   
 $A \rightarrow aA | a$ ,  $B \rightarrow bB | \epsilon$ ,  $C \rightarrow C$

2) To eliminate  $B \rightarrow \epsilon$

$$S \rightarrow AAC | AC | C, \quad B \rightarrow b$$

New production:  $S \rightarrow ABAC | ABC | BAC | BC | AAC | AC | C$   
 $A \rightarrow aA | a$   
 $B \rightarrow bB | b$   
 $C \rightarrow C$

Questions???