L4.2 Equivalence of CFG and PDA (Part 2)

Syllabus Topic
PDA and Context Free Language, Equivalence of PDA and CFG

Pushdown Automata and Context Free Language

The class of languages accepted by pushdown automata is exactly the class of context-free languages. The following three classes of languages are same :

1. Context Free Language defined by CFG.
2. Languages accepted by PDA by final state.
3. Languages accepted by PDA by empty stack.

- It is possible to find a PDA for a CFG
- It is possible to find a CFG for a PDA.

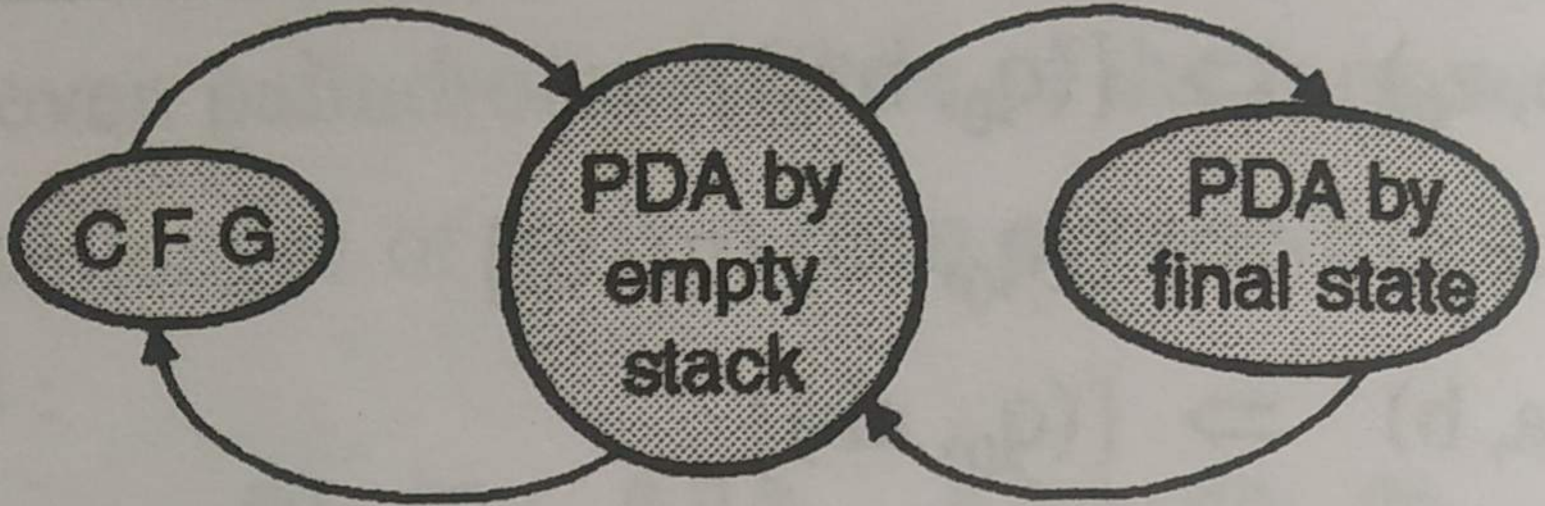

Fig. 5.6.1 : Equivalence of PDA and CFG

5.6.1 Construction of PDA from CFG

From a given CFG G = (V, T, P, S), we can construct a PDA, M that simulates the leftmost derivation of G.

The PDA accepting L(G) by empty stack is given by :

$$M = (\{q\}, T, V \cup T, \delta, q, S, \phi)$$

[M is a PDA for L(G)]

Where δ is defined by :

1. For each variable $A \in V$, include a transition,

   $\delta(q, \varepsilon, A) \Rightarrow \{(q, \alpha) \mid A \to \alpha \text{ is a production in } G\}$

2. For each terminal $a \in T$, include a transition

   $\delta(q, a, a) \Rightarrow \{(q, \varepsilon)\}$

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Example 5.6.1

Find a PDA for the given grammar

$S \rightarrow 0S1 \mid 00 \mid 11$

Solution : The equivalent PDA, M is given by :

$$M = (\{q\}, \{0, 1\}, \{0, 1, S\}, \delta, q, S, \phi),$$

where δ is given by :

$$\delta(q, \varepsilon, S) = \{(q, 0S1), (q, 00), (q, 11)\}$$

$$\delta(q, 0, 0) = \{(q, \varepsilon)\}$$

$$\delta(q, 1, 1) = \{(q, \varepsilon)\}$$

Example 5.6.2 SPPU - May 12, 5 Marks

Convert the grammar $S \rightarrow 0S1|A \quad A \rightarrow 1A0 \mid S \mid \varepsilon$ to PDA that accepts the same language by empty stack.

Solution :

Step 1 : For each variable $A \in V$, include a transition

$\delta(q, \varepsilon, A) \Rightarrow \{(q, \alpha) \mid A \rightarrow \alpha$ is a production in G $\}$

$$\delta(q, \varepsilon, S) \Rightarrow \{(q, 0S1), (q, A)\}$$

$$\delta(q, \varepsilon, A) \Rightarrow \{(q, 1A0), (q, S), (q, \varepsilon)\}$$

Step 2 : For each terminal $a \in T$, include a transition

$\delta(q, a, a) \Rightarrow (q, \varepsilon)$

$$\therefore \quad \delta(q, 0, 0) = \{(q, \varepsilon)\}$$

$$\delta(q, 1, 1) = \{(q, \varepsilon)\}$$

Therefore, the PDA is given by :

$M = (\{q\}, \{0, 1\}, \{S, A, 0, 1\}, \delta, q, S, \phi)$

where δ is : $\delta(q, \varepsilon, S) = \{(q, 0S1), (q, A)\}$

$$\delta(q, \varepsilon, A) = \{(q, 1A0), (q, S), (q, \varepsilon)\}$$

$$\delta(q, 0, 0) = \{(q, \varepsilon)\}$$

$$\delta(q, 1, 1) = \{(q, \varepsilon)\}$$

Example 5.6.3

Let G be the grammar given by S →aABB | aA
A → aBB | a, B → bBB | A. Construct NPDA th
accepts the language generated by this grammar.

Solution : The equivalent PDA, M is given by :

$$M = (\{q\}, \{a, b\}, \{a, b, S, A, B\}, \delta, q, S, \phi)$$

where δ is given by :

$\delta(q, \varepsilon, S) \Rightarrow \{(q, aABB), (q, aAA)\}$
$\delta(q, \varepsilon, A) \Rightarrow \{(q, aBB), (q, a)\}$
$\delta(q, \varepsilon, B) \Rightarrow \{(q, bBB), (q, A)\}$

For each producti
in given gramm

$\delta(q, a, a) \Rightarrow (q, \varepsilon)$
$\delta(q, b, b) \Rightarrow (q, \varepsilon)$

For each terminal in T.

Example 5.6.4 SPPU - May 12, May 13, 5 Marks

Construct a PDA equivalent to the following CFG. $S \rightarrow 0BB \quad B \rightarrow 0S \mid 1S \mid 0$ Test if 010^4 is in the language.

Solution : The equivalent PDA, M is given by

$$M = (\{q\},\{0, 1\},\{0, 1, S, B\}, \delta, q, S, \phi),$$

where δ is given by :

$\delta(q, \varepsilon, S) \Rightarrow \{(q, 0BB)\}$
$\delta(q, \varepsilon, B) \Rightarrow \{(q, 0S), (q, 1S), (q, 0)\}$ — For each production in the given grammar

$\delta(q, 0, 0) \Rightarrow \{(q, \varepsilon)\}$
$\delta(q, 1, 1) \Rightarrow \{(q, \varepsilon)\}$ — For each terminal.

Acceptance of 010^4 by M

$$\delta(q, 010000, S) \xrightarrow{\delta(q, \varepsilon, S) = (q, 0BB)} (q, 010000, 0BB)$$

$$\xrightarrow{\delta(q, 0, 0) = (q, \varepsilon)} (q, 10000, BB)$$

$$\xrightarrow{\delta(q, \varepsilon, B) = (q, 1S)} (q, 10000, 1SB)$$

$$\xrightarrow{\delta(q, 1, 1) = (q, \varepsilon)} (q, 0000, SB)$$

$$\xrightarrow{\delta(q, \varepsilon, S) = (q, 0BB)} (q, 0000, 0BBB)$$

$$\xrightarrow{\delta(q, 0, 0) = (q, \varepsilon)} (q, 000, BBB)$$

$$\xrightarrow{\delta(q, \varepsilon, B) = (q, 0)} (q, 000, 0BB)$$

$$\xrightarrow{\delta(q, 0, 0) = (q, \varepsilon)} (q, 00, BB)$$

$$\xrightarrow{\delta(q, \varepsilon, B) = (q, 0)} (q, 00, 0B)$$

$$\xrightarrow{\delta(q, 0, 0) = (q, \varepsilon)} (q, 0, B)$$

$$\xrightarrow{\delta(q, \varepsilon, B) = (q, 0)} (q, 0, 0)$$

$$\xrightarrow{\delta(q, 0, 0) = (q, \varepsilon)} (q, \varepsilon, \varepsilon)$$

Thus the string 010^4 is accepted by M using an empty stack.

$$\therefore \; 010^4 \in L$$

Example 5.6.5

Design a PDA to recognize the language generated by the following grammar :

$S \rightarrow S + S \mid S * S \mid 4 \mid 2$

Show the acceptance of the input string 2 + 2 * 4 by this PDA.

Solution : The equivalent PDA, M is given by :

$M = (\{q\}, \{+, *, 4, 2\}, \{+, *, 4, 2, S\}, \delta, q, S, \phi)$

where δ is given by :

$\delta(q, \varepsilon, S) \Rightarrow \{(q, S + S), (q, S * S), (q, 4), (q, 2)\}$ | for every production in G

$\delta(q, +, +) = \{(q, \varepsilon)\}$
$\delta(q, *, *) = \{(q, \varepsilon)\}$
$\delta(q, 2, 2) = \{(q, \varepsilon)\}$
$\delta(q, 4, 4) = \{(q, \varepsilon)\}$

for every terminal in T.

Acceptance of 2 + 2 * 4 by this PDA

$\delta(q, 2 + 2 * 4, S) \xrightarrow{\delta(q, \varepsilon, S) \Rightarrow (q, S + S)} (q, 2 + 2 * 4, S + S)$

$\xrightarrow{\delta(q, \varepsilon, S) \Rightarrow (q, 2)} (q, 2 + 2 * 4, 2 + S)$

$\xrightarrow{\delta(q, 2, 2) \Rightarrow (q, \varepsilon)} (q, + 2 * 4, + S)$

$\xrightarrow{\delta(q, +, +) \Rightarrow (q, \varepsilon)} (q, 2 * 4, S)$

$\xrightarrow{\delta(q, \varepsilon, S) \Rightarrow (q, S * S)} (q, 2 * 4, S * S)$

$\xrightarrow{\delta(q, \varepsilon, S) \Rightarrow (q, 2)} (q, 2 * 4, 2 * S)$

$\xrightarrow{\delta(q, 2, 2) \Rightarrow (q, \varepsilon)} (q, * 4, * S)$

$\xrightarrow{\delta(q, *, *) \Rightarrow (q, \varepsilon)} (q, 4, S)$

$\xrightarrow{\delta(q, \varepsilon, S) = (q, 4)} (q, 4, 4)$

$\xrightarrow{\delta(q, 4, 4) = (q, \varepsilon)} (q, \varepsilon, \varepsilon)$