L5 PDA to CFG

5.6.2 Construction of CFG from PDA

SPPU - May 12

University Question

Q. Describe in brief : Construction of CFG from PDA. **(May 2012, 5 Marks)**

We can find the Context Free Grammar G for any PDA, M such that

$$L(G) = L(M)$$

i.e., we can construct an equivalent CFG for a PDA.

The variables of the CFG, so constructed will be of the form :

$$[p^X q], \text{ where } p, q \in Q \text{ and } X \in \Gamma$$

Let the PDA is given by :

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z, \phi)$$

where z is the initial stack symbol.

Then an equivalent CFG is given by

$$G = (V, \Sigma, P, S) \text{ where}$$

$$V = \{S, [p^X q] \mid p, q \in Q \text{ and } X \in \Gamma\}$$

Example : If $Q = \{q_0, q_1\}$ and $\Gamma = \{a, b, z\}$ then the possible set of variables in the corresponding CFG is given by :

1. S
2. $[q_0\ ^{a}\ q_0], [q_0\ ^{a}\ q_1], [q_1\ ^{a}\ q_0], [q_1\ ^{a}\ q_1]$
3. $[q_0\ ^{b}\ q_0], [q_0\ ^{b}\ q_1], [q_1\ ^{b}\ q_0], [q_1\ ^{b}\ q_1]$
4. $[q_0\ ^{z}\ q_0], [q_0\ ^{z}\ q_1], [q_1\ ^{z}\ q_0], [q_1\ ^{z}\ q_1]$

Set of productions for the equivalent CFG

1. Add the following productions for the start symbol S.

   $S \rightarrow [q_0\ ^{z}\ q_i]$ for each $q_i \in Q$, where z is the start symbol

2. For each transition of the form

   $$\delta (q_i, a, B) \Rightarrow (q_j, C)$$

   where,

   (a) $q_i, q_j \in Q$ (b) a belongs to $(\Sigma \cup \varepsilon)$

   (c) B and C belong to $(\Gamma \cup \varepsilon)$

   Then for each $q \in Q$, we add the production :

   $$[q_i\ ^{B}\ q] \rightarrow a\ [q_j\ ^{C}\ q]$$

3. For each transition of the form

   $$\delta (q_i, a, B) \Rightarrow (q_j, C_1\ C_2)$$

   where,

   (a) $q_i, q_j \in Q$

(b) a belongs to $(\Sigma \cup \varepsilon)$

(c) B, C_1 and C_2 belongs to Γ

then for each $p_1, p_2 \in Q$, we add the production

$$[q_i \overset{B}{} p_1] \rightarrow a [q_j \overset{C_1}{} p_2] [p_2 \overset{C_2}{} p_1]$$

Convert PDA to CFG. PDA is given by

M = ({ p, q}, {0, 1}, {x, z}, δ, q, z), transition function δ is defined by :

$\delta(q, 1, z) \Rightarrow \{(q, xz)\}$

$\delta(q, 1, x) \Rightarrow \{(q, xx)\}$

$\delta(q, \varepsilon, x) \Rightarrow \{(q, \varepsilon)\}$

$\delta(q, 0, x) \Rightarrow \{(p, x)\}$

$\delta(p, 1, x) \Rightarrow \{(p, \varepsilon)\}$

$\delta(p, 0, z) \Rightarrow \{(q, z)\}$

Solution :

Step 1 : Add productions for the start symbol.

$S \rightarrow [q\ ^{z}\ q]$

$S \rightarrow [q\ ^{z}\ p]$

Step 2 : Add productions for

$\delta(q, 1, z) \Rightarrow \{(q, xz)\}$

$[q\ ^{z}\ q] \rightarrow 1\ [q\ ^{x}\ q]\ [q\ ^{z}\ q]$

$[q\ ^{z}\ q] \rightarrow 1\ [q\ ^{x}\ p]\ [p\ ^{z}\ q]$

$[q\ ^{z}\ p] \rightarrow 1\ [q\ ^{x}\ q]\ [q\ ^{z}\ p]$

$[q\ ^{z}\ p] \rightarrow 1\ [q\ ^{x}\ p]\ [p\ ^{z}\ p]$

Step 3 : Add productions for

Step 4 : Add productions for

(b) a belongs to $(\sum \cup \varepsilon)$

(c) B, C_1 and C_2 belongs to Γ

then for each $p_1, p_2 \in Q$, we add the production

$$[q_i\ ^{B}\ p_1] \rightarrow a\ [q_j\ ^{C_1}\ p_2]\ [p_2\ ^{C_2}\ p_1]$$

Step 3 : Add productions for

$\delta(q, 1, x) \Rightarrow \{(q, xx)\}$

$[q\ ^{x}\ q] \rightarrow 1\ [q\ ^{x}\ q]\ [q\ ^{x}\ q]$

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$[q\ ^{x}\ p] \rightarrow 1\ [q\ ^{x}\ p]\ [p\ ^{x}\ p]$

Step 4 : Add productions for

$\delta(q, \varepsilon, x) \Rightarrow \{(q, \varepsilon)\}$

$[q\ ^{x}\ q] \rightarrow \varepsilon$

Step 5 : Add productions for

$\delta(q, 0, x) \Rightarrow \{(p, x)\}$

$[q\ ^{x}\ q] \rightarrow 0\ [p\ ^{x}\ q]$

$[q\ ^{x}\ p] \rightarrow 0\ [p\ ^{x}\ p]$

Step 6 : Add productions for

$\delta(p, 1, x) \Rightarrow \{(p, \varepsilon)\}$

$[p\ ^{x}\ p] \rightarrow 1$

Step 7 : Add productions for $\delta(p, 0, z) \Rightarrow \{(q, z)\}$

$[p\ ^{z}\ q] \rightarrow 0\ [q\ ^{z}\ q]$

$[p\ ^{z}\ p] \rightarrow 0\ [q\ ^{z}\ p]$

Example 5.6.7 SPPU - Dec. 12, 10 Marks

Convert PDA to CFG. PDA is given by

M = ({ p, q}, {0, 1}, {x, z}, δ, q, z), transition function δ is defined by :

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$\delta(p, 1, x) \Rightarrow \{(p, \varepsilon)\}$

$\delta(p, 0, z) \Rightarrow \{(q, z)\}$

Solution :

Step 1 : Add productions for the start symbol.

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$S \rightarrow [q\ ^{z}\ p]$

Step 2 : Add productions for

$\delta(q, 1, z) \Rightarrow \{(q, xz)\}$

$[q\ ^{z}\ q] \rightarrow 1\ [q\ ^{x}\ q]\ [q\ ^{z}\ q]$

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$[q\ ^{x}\ p] \rightarrow 1\ [q\ ^{x}\ p]\ [p\ ^{x}\ p]$

Step 4 : Add productions for

$\delta(q, \varepsilon, x) \Rightarrow \{(q, \varepsilon)\}$

$[q\ ^{x}\ q] \rightarrow \varepsilon$

Step 5 : Add productions for

$\delta(q, 0, x) \Rightarrow \{(p, x)\}$

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Example 5.6.7 SPPU - Dec. 12, 10 Marks

Convert PDA to CFG. PDA is given by

M = ({ p, q}, {0, 1}, {x, z}, δ, q, z), transition function δ is defined by :

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$\delta(p, 1, x) \Rightarrow \{(p, \varepsilon)\}$

$\delta(p, 0, z) \Rightarrow \{(q, z)\}$

Solution :

Step 1 : Add productions for the start symbol.

$S \rightarrow [q^z q]$

$S \rightarrow [q^z p]$

Step 2 : Add productions for

$\delta(q, 1, z) \Rightarrow \{(q, xz)\}$

$[q^z q] \rightarrow 1\,[q^x q]\,[q^z q]$

$[q^z q] \rightarrow 1\,[q^x p]\,[p^z q]$

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$\delta(q, 1, x) \Rightarrow \{(q, xx)\}$

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$\delta(q, \varepsilon, x) \Rightarrow \{(q, \varepsilon)\}$

$[q^x q] \rightarrow \varepsilon$

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Step 7 : Add productions for $\delta(p, 0, z) \Rightarrow \{(q, z)\}$

$[p^z q] \rightarrow 0\,[q^z q]$

$[p^z p] \rightarrow 0\,[q^z p]$

Step 8 : Renaming of variables :

Original name	New name
$[q^z q]$	A
$[q^z p]$	B
$[p^z q]$	C
$[p^z p]$	D
$[q^x q]$	E
$[q^x p]$	F
$[p^x q]$	G
$[p^x p]$	H

The set of productions can be written as :

$S \rightarrow A \mid B$

$A \rightarrow 1EA \mid 1FC$

$B \rightarrow 1EB \mid 1FD$

$E \rightarrow 1EE \mid 1FG$

$F \rightarrow 1EF \mid 1FH$

$E \rightarrow \varepsilon$

$E \rightarrow 0G$

$F \rightarrow 0H$

$H \rightarrow 1$

$C \rightarrow 0A$

$D \rightarrow 0B$

Step 9 : Simplification of grammar

Symbol G does not come on the left side of the production, hence it can be eliminated.

The equivalent set of productions is :

$S \rightarrow A \mid B$

$A \rightarrow 1EA \mid 1FC$

$B \rightarrow 1EB \mid 1FD$

$E \rightarrow 1EE \mid \varepsilon$

$F \rightarrow 1EF \mid 1FH \mid 0H$

$H \rightarrow 1$

$C \rightarrow 0A$

$D \rightarrow 0B$

Example 5.6.8 SPPU - Dec. 13, May 15, May 16